Models and data combined to progress towards a better understanding of the magnetism of solar-type stars

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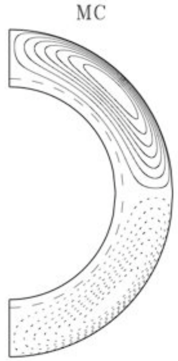
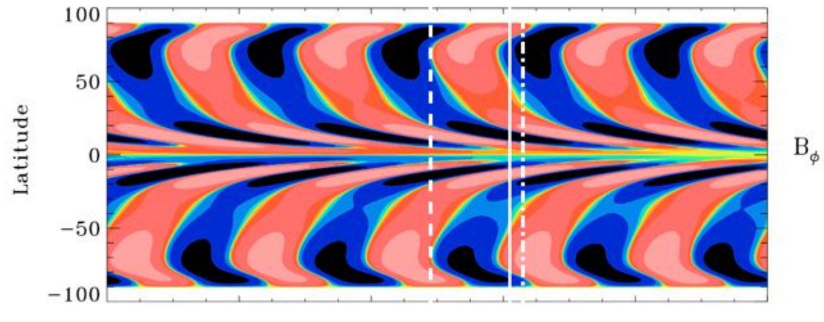
Some open questions about the solar/stellar internal magnetism

- ❑ What sets the magnetic cycle period, field topology and strength?
- ❑ What is the role of the stellar structure?
- ❑ What is the role of mean flows (rotation and meridional flow)?
- ❑ How does the magnetic field emerge at the stellar surface?
- ❑ Are sunspots/starspots a necessary ingredient of the dynamo?
- ❑ What is the structure of spots? How is it created?
- ❑ Do small-scale and large-scale dynamos interact? How?
- ❑ What is the role of the magnetic helicity?
- ❑ Can we predict future solar/stellar activity?
- ❑ …

The Sun: meridional flow internal profile

- Some dynamo models (Babcock-Leighton flux transport) using 1 single cell per hemisphere produce butterfly diagrams in agreement with observations



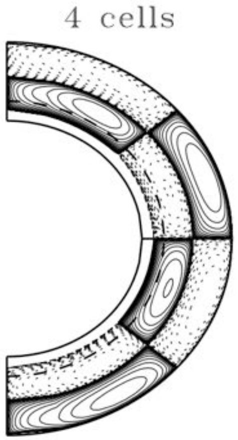


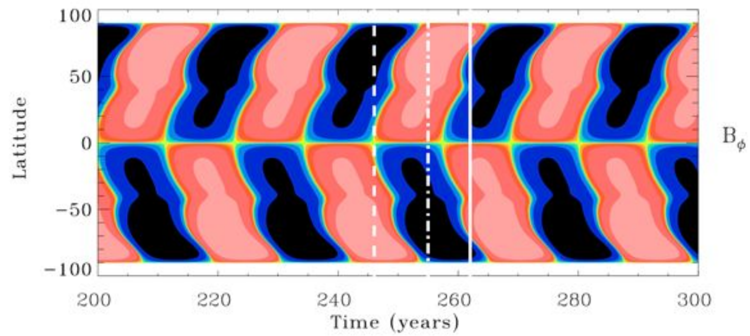
- BUT from observations and simulations, the MC may be multicellular

- If a complex profile persists for the whole cycle, the effect on the magnetic field may be dramatic

Jouve & Brun, 2007

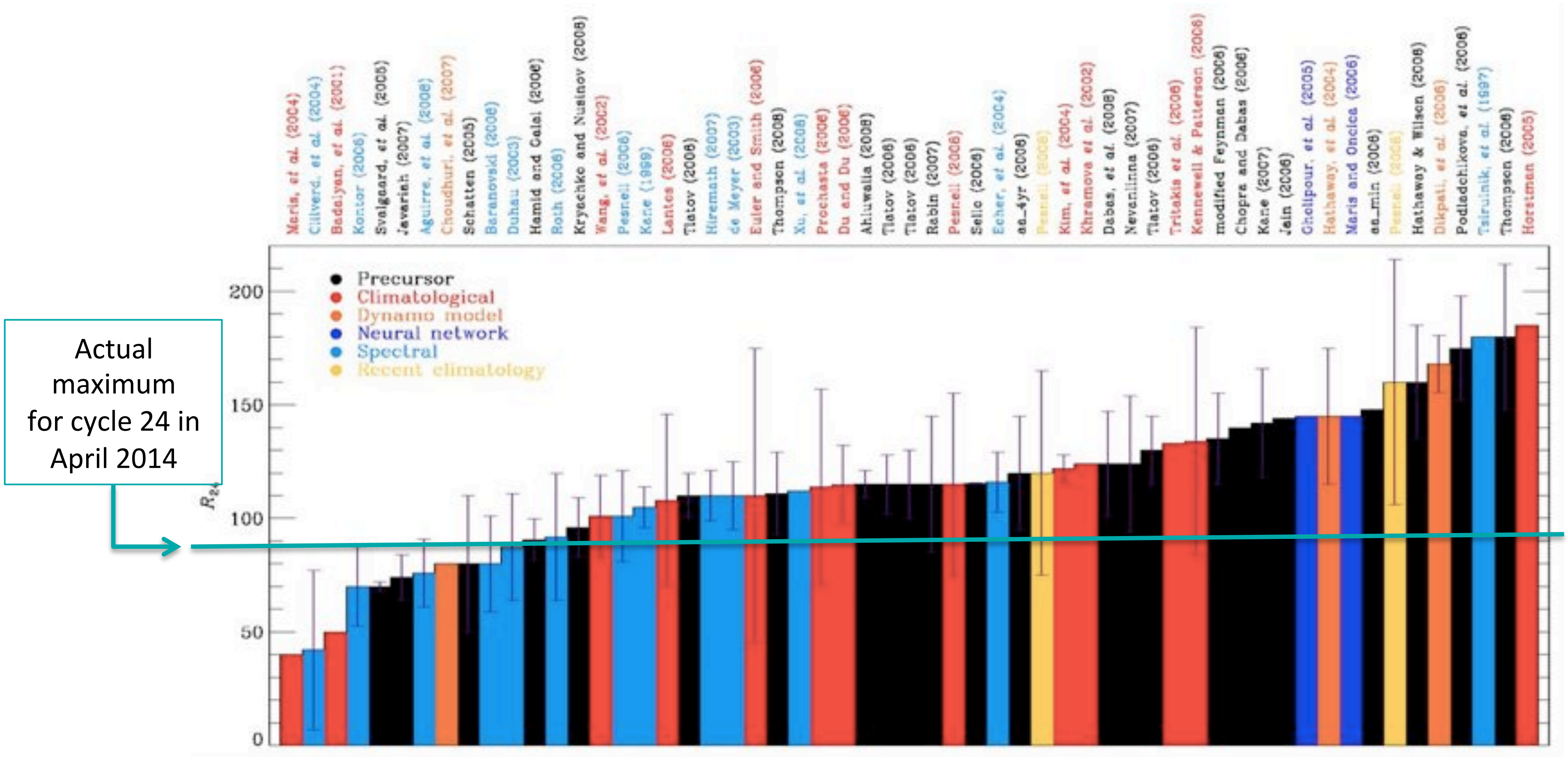
BL model:
MC with 4 cells per hemisphere





Butterfly diagram no longer in agreement with observations

The Sun: predicting future solar activity?

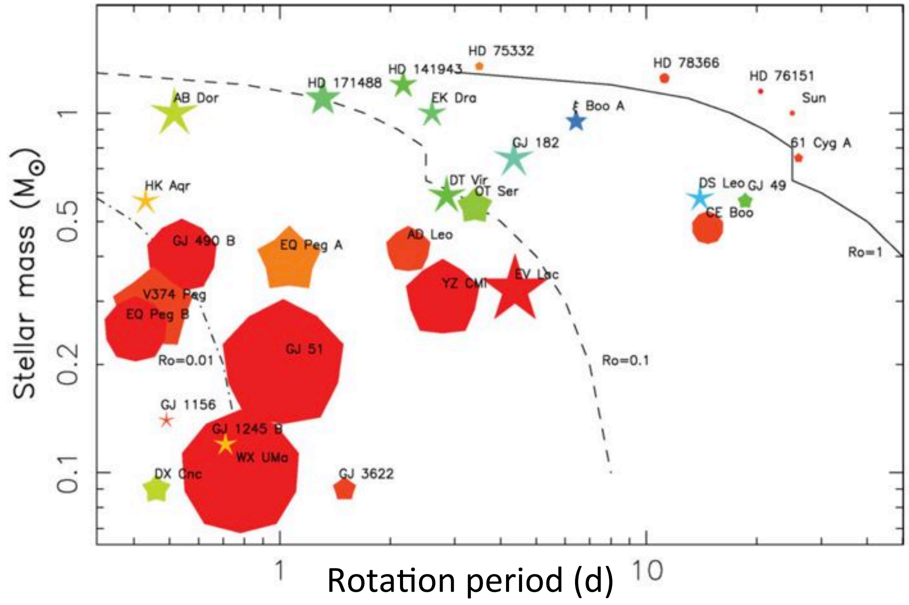


Pesnell 2012

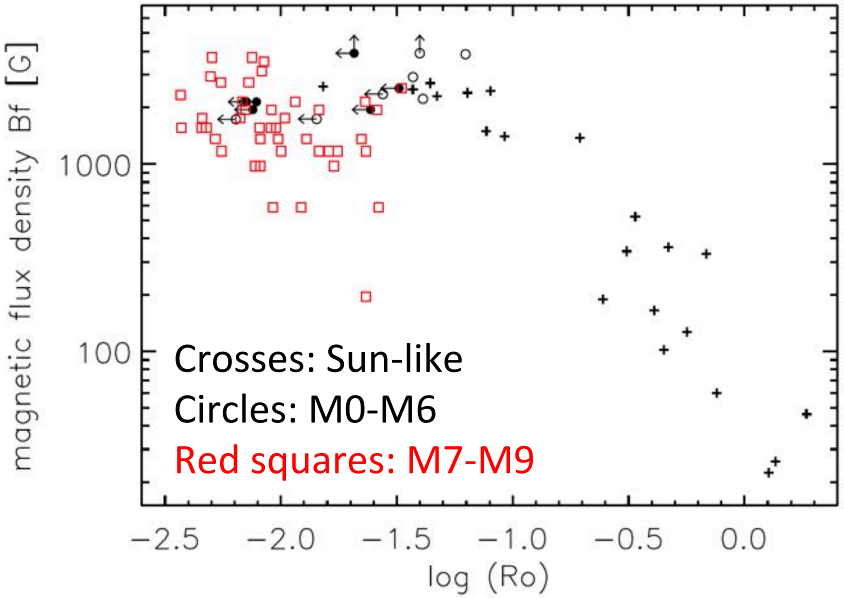
Tools to answer those questions

❑ Observations:
Spectropolarimetry, seismology, vector magnetograms, activity measurements,…

Morin et al. 2010, Donati 2011

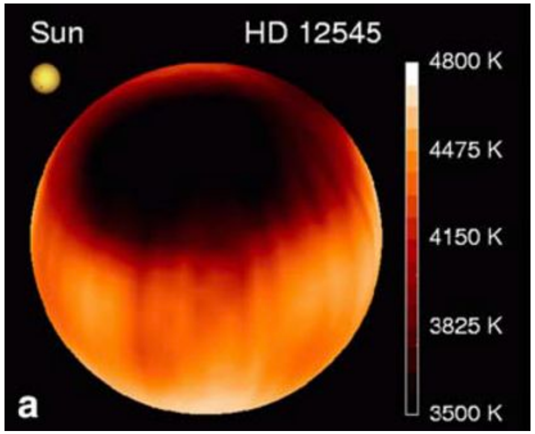


Reiners (2012)



Rossby number=P_{rot}/τ_c
Increase of field strength with rotation
Saturation for fast rotation

Strassmeier (1999)



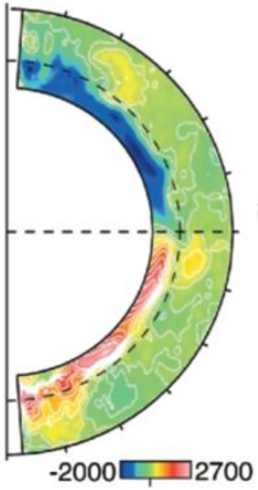
Spots detected in stars cooler than the Sun, covering large fraction of the stellar surface

Tools to answer those questions

- **Theory/models:**

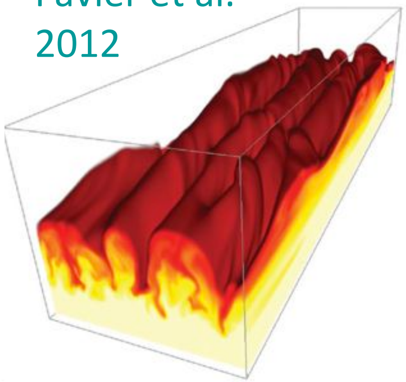
Low-order models, local 2D/3D simulations, global 2D/3D simulations,…

Flux emergence from tachocline

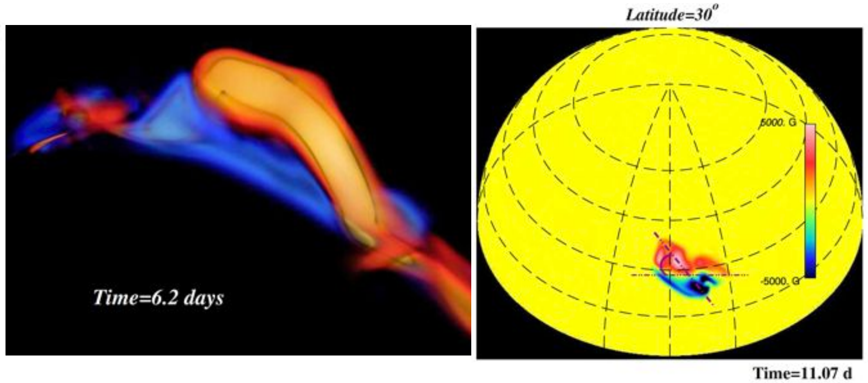


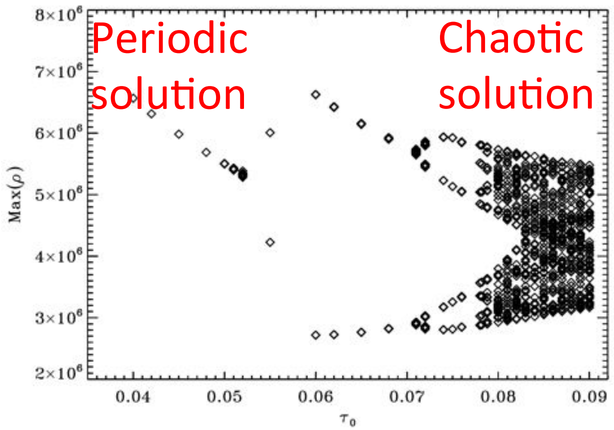
Browning et al. 2006

Favier et al. 2012



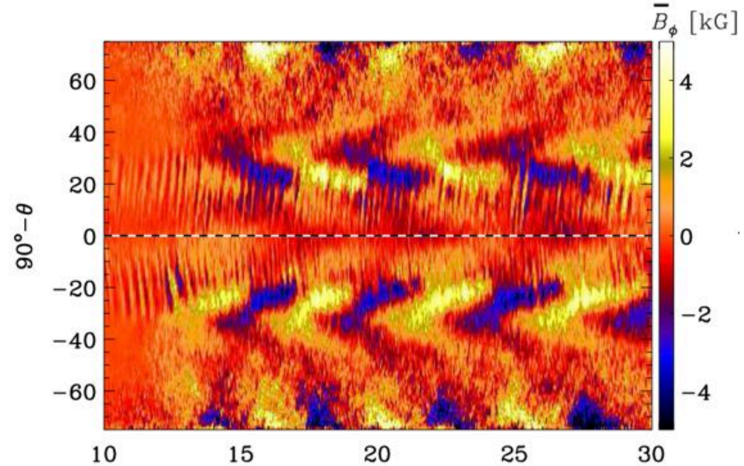
Jouve et al. 2013





Low order models

Jouve et al. 2010



Equatorward dynamo wave

Warnecke et al. 2014

Combining data and models

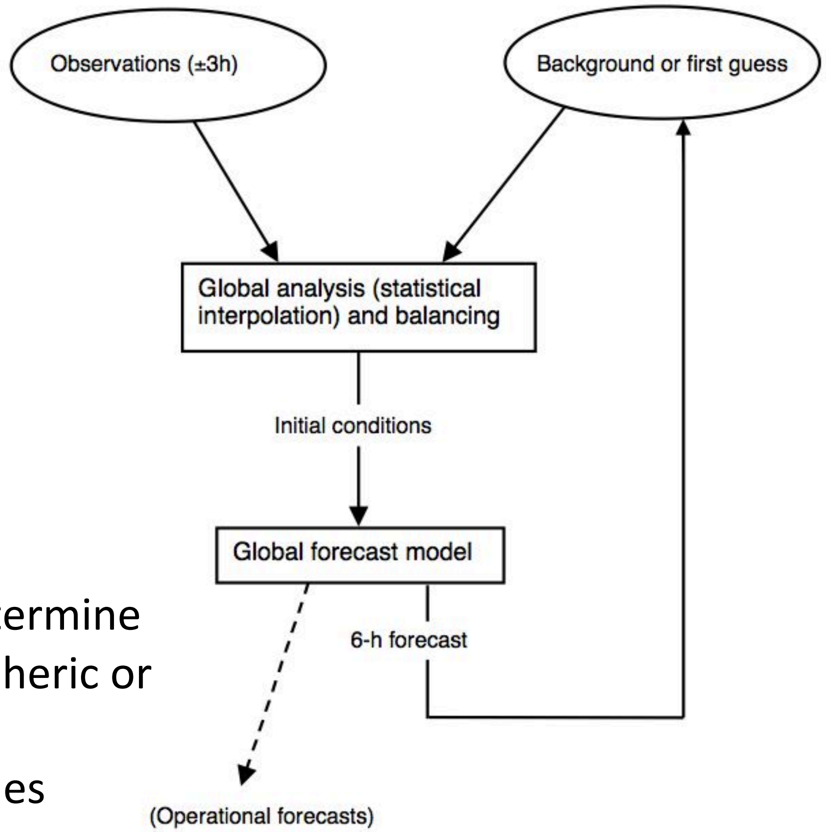
❑ Direct comparison of model outputs with observations

❑ Drive models with data:

- Magnetic observations into dynamo models (Dikpati et al. 2006, Choudhuri et al. 2007)
- Active regions into surface flux-transport (Schrijver & DeRosa 2003, Cheung & DeRosa 2012, P. Bhowmik's poster)

❑ Assimilate data into models:

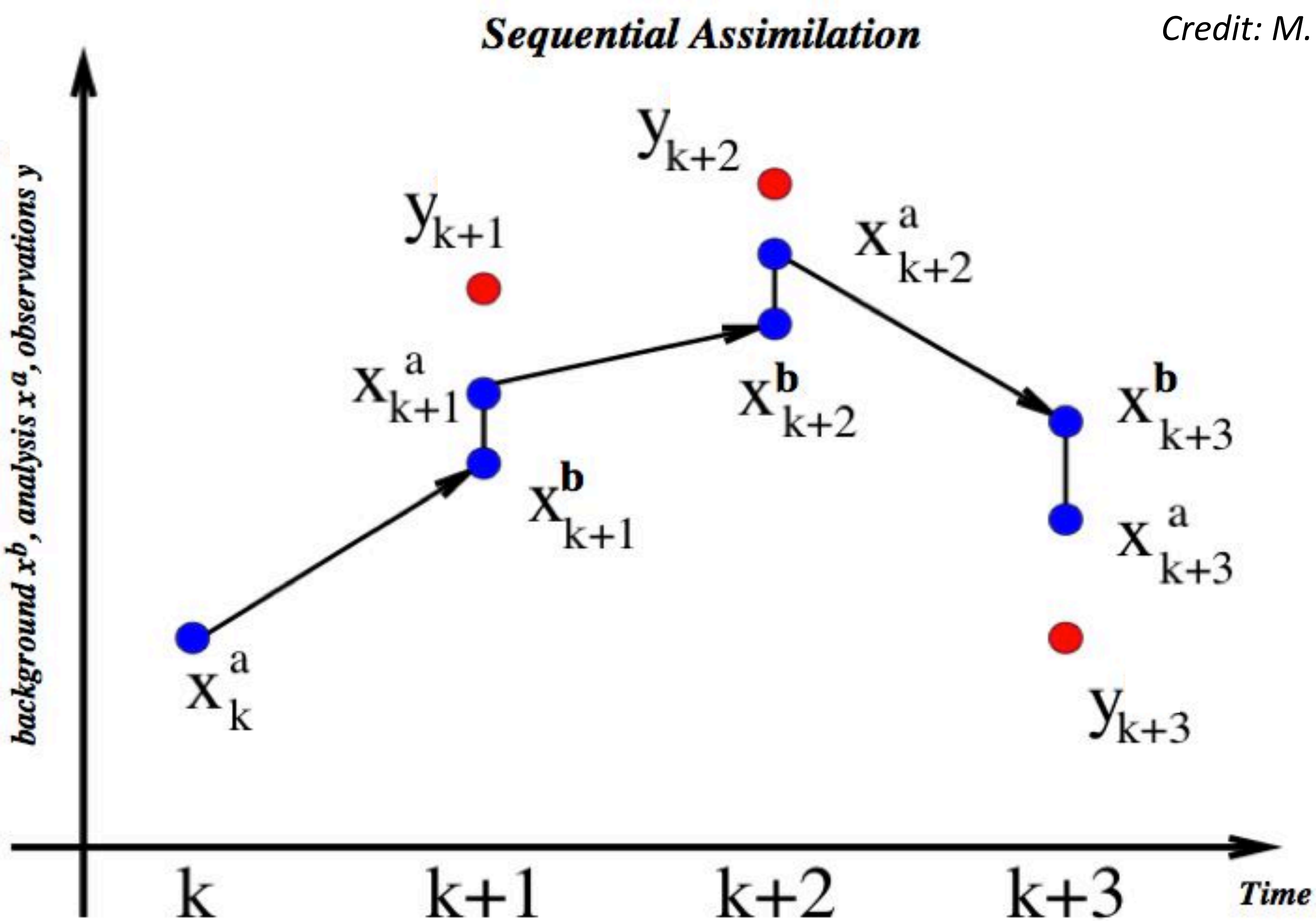
- Purpose: using all available information to determine as accurately as possible the state of the atmospheric or oceanic flow (Talagrand, 1997)
- Operational for weather forecasting for decades



Credit: E. Kalnay

Sequential data assimilation

Credit: M. Bocquet



Propagates information forward in time

Sequential assimilation applied to solar physics

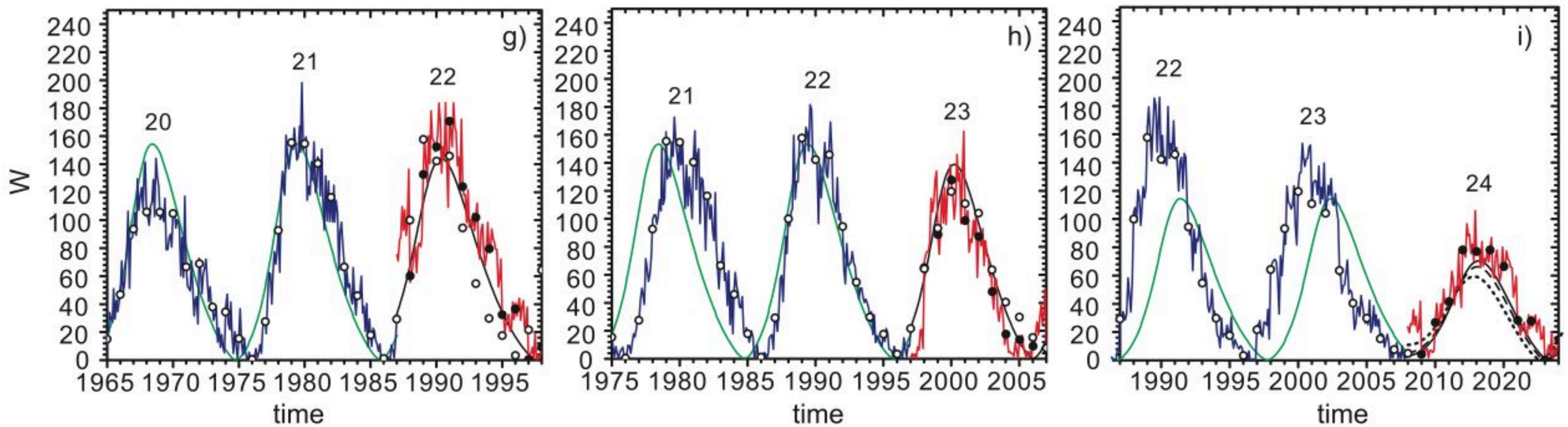
- Kalman filter method was used by Kitiashvili et al. (2008) on an $\alpha\Omega$ dynamo model.

$$\frac{dA}{dt} = \alpha B - \eta k^2 A, \quad \frac{dB}{dt} = ikGA - \eta k^2 B,$$

$$\frac{d\alpha_m}{dt} = -\frac{\alpha_m}{T} - \frac{Q}{2\pi\rho}\left[-ABk^2 + \frac{\alpha}{\eta}(B^2 - k^2 A^2)\right]$$

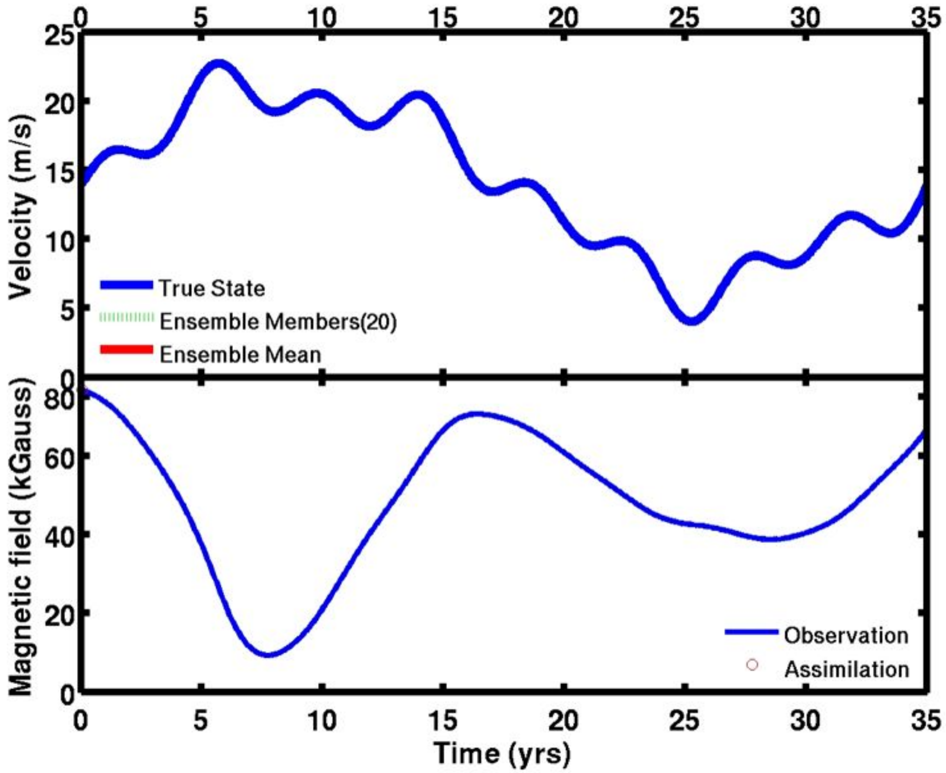
- Sunspot numbers are assimilated into the model.

- Forecast step is tested on previous cycles and produced for cycle 24.



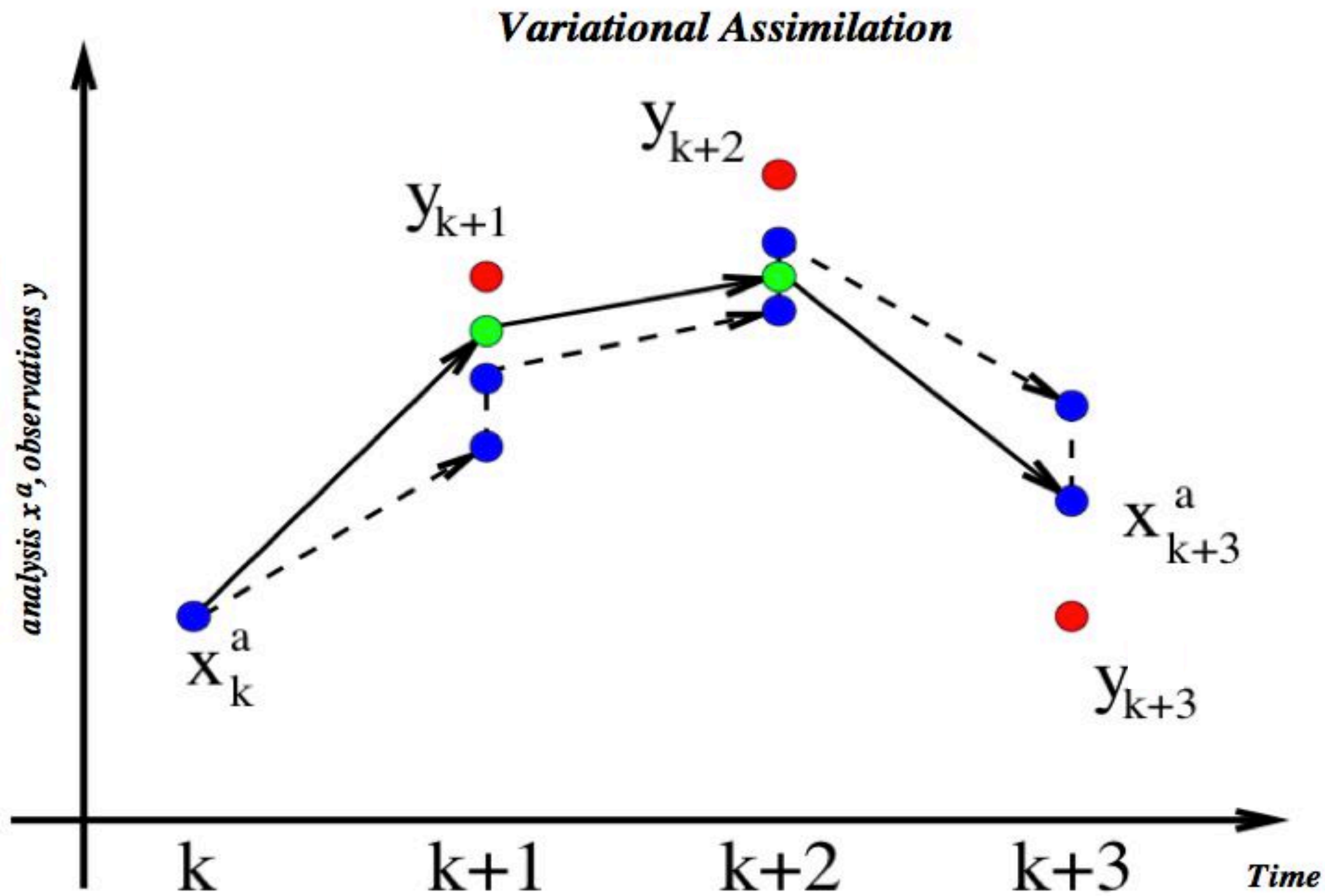
Sequential assimilation applied to solar physics

- Ensemble Kalman filter method is used by Dikpati et al. (2014) on a BL dynamo model.
- Magnetic field intensity (toroidal field here) is assimilated into the model.
- They reconstruct the amplitude of the meridional flow.



Credit: M. Dikpati

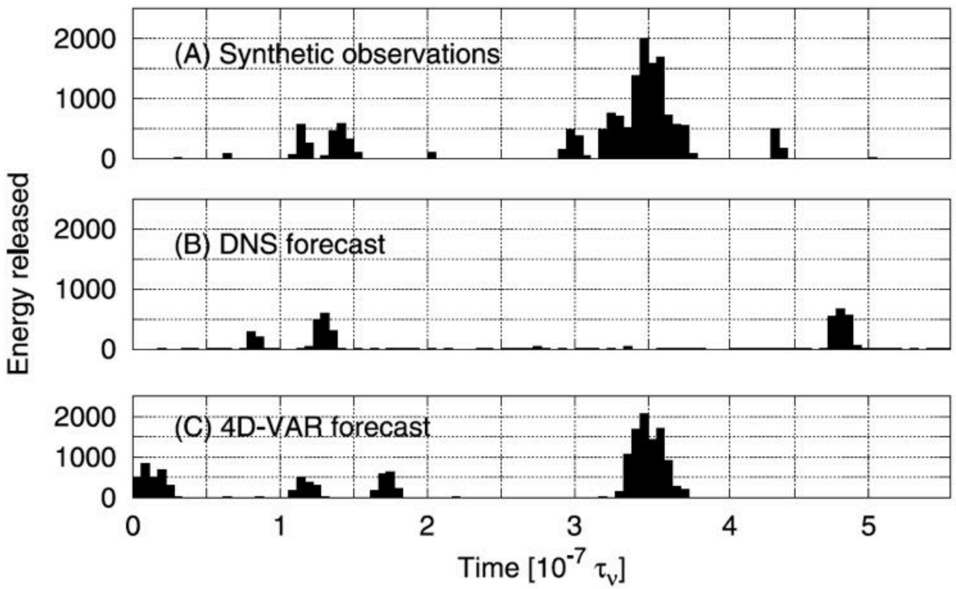
Variational data assimilation

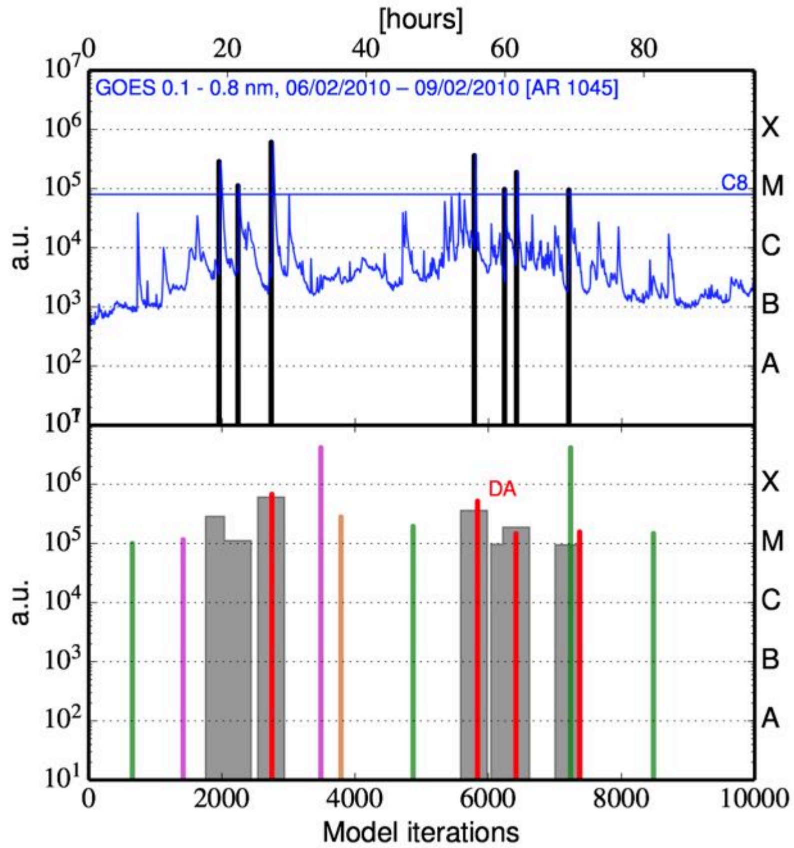


Propagates information both forward and backward in time

Variational assimilation applied to solar physics

❑ 4D-Var applied in model of solar flares, use of synthetic data (Bélanger et al. 2007) and GOES data with modified version of avalanche model (Strugarek & Charbonneau 2014)





❑ 4D-Var applied to a simple $\alpha\Omega$ model in Cartesian geometry (Jouve, Brun & Talagrand 2011)

4DVar in a dynamo model:

BLFT model in spherical geometry

Hung, Jouve, Brun, Fournier & Talagrand, 2015

- Model equations

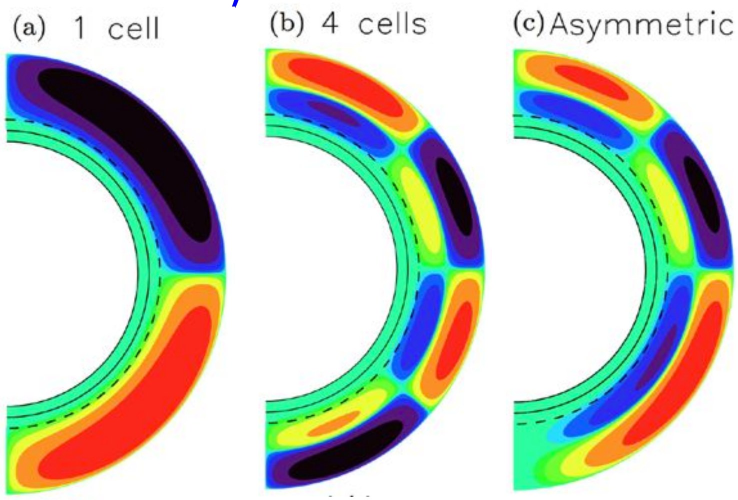
$$\partial_t A_\phi = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right) A_\phi - Re\frac{\vec{v}_p}{\varpi}\cdot\nabla(\varpi A_\phi) + C_s S(r,\theta,B_\phi),$$

$$\partial_t B_\phi = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right) B_\phi + \frac{1}{\varpi}\frac{\partial(\varpi B_\phi)}{\partial r}\frac{\partial(\eta/\eta_t)}{\partial r} - Re\varpi\vec{v}_p\cdot\nabla\left(\frac{B_\phi}{\varpi}\right) - ReB_\phi\nabla\cdot\vec{v}_p + C_\Omega\varpi\left[\nabla\times(A_\phi\hat{e}_\phi)\right]\cdot\nabla\Omega,$$

- Meridional circulation: the main ingredient (control variable)

$$\vec{v}_p = \nabla\times(\psi\hat{e}_\phi)$$

$$\psi(r,\theta) = -\frac{2(r-r_{mc})^2}{\pi(1-r_{mc})} \times \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n} d_{i,j}\sin\left[\frac{i\pi(r-r_{mc})}{1-r_{mc}}\right]P_j^1(-\cos\theta) & \text{if } r_{mc}\le r\le 1 \\ 0 & \text{if } r_{bot}\le r< r_{mc}; \end{cases}$$



m=2, n=4 => 8 coefs $d_{i,j}$

Case	$d_{1,2}$	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$	$d_{2,4}$
1	3.33×10^{-1}	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	9.47×10^{-2}
3	0.00	-5.74×10^{-2}	-8.75×10^{-2}	-3.83×10^{-2}	5.47×10^{-2}

4DVar in a dynamo model:
Twin experiments

- We produce synthetic observations with a given MC and input parameters

- We noise the data (normal distribution with std=percentage of σ)

- We choose a cost function to be minimized:

$$\mathcal{J}_A = \sum_{i=1}^{N_t^o} \sum_{j=1}^{N_\theta^o} \frac{\left[A_\phi(R_s,\theta_j,t_i) - A_\phi^o(R_s,\theta_j,t_i)\right]^2}{\sigma_{A_\phi}^2(R_s,\theta_j)}, \quad \mathcal{J}_B = \sum_{i=1}^{N_t^o} \sum_{j=1}^{N_\theta^o} \frac{\left[B_\phi(r_c,\theta_j,t_i) - B_\phi^o(r_c,\theta_j,t_i)\right]^2}{\sigma_{B_\phi}^2(r_c,\theta_j)},$$

- Initial guess for the minimization procedure: a 1 cell MC
- Initial conditions for direct code: magnetic field produced by this 1 cell model

- We minimize the cost function (using J and its gradient) by adjusting the control vector $d_{i,j}$

- The diagnostic quantities:

$$\frac{\Delta p}{p} = \sqrt{\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(d_{i,j} - d_{i,j\,true})^2}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{i,j\,true}^{\;2}}}, \qquad \mathcal{J}/\mathcal{J}_o \quad \text{and} \quad \mathcal{J}_{norm} = \frac{1}{\epsilon}\sqrt{\frac{\mathcal{J}}{N}}, \quad \text{when noisy data}$$

4DVar in a dynamo model:

Results without noise

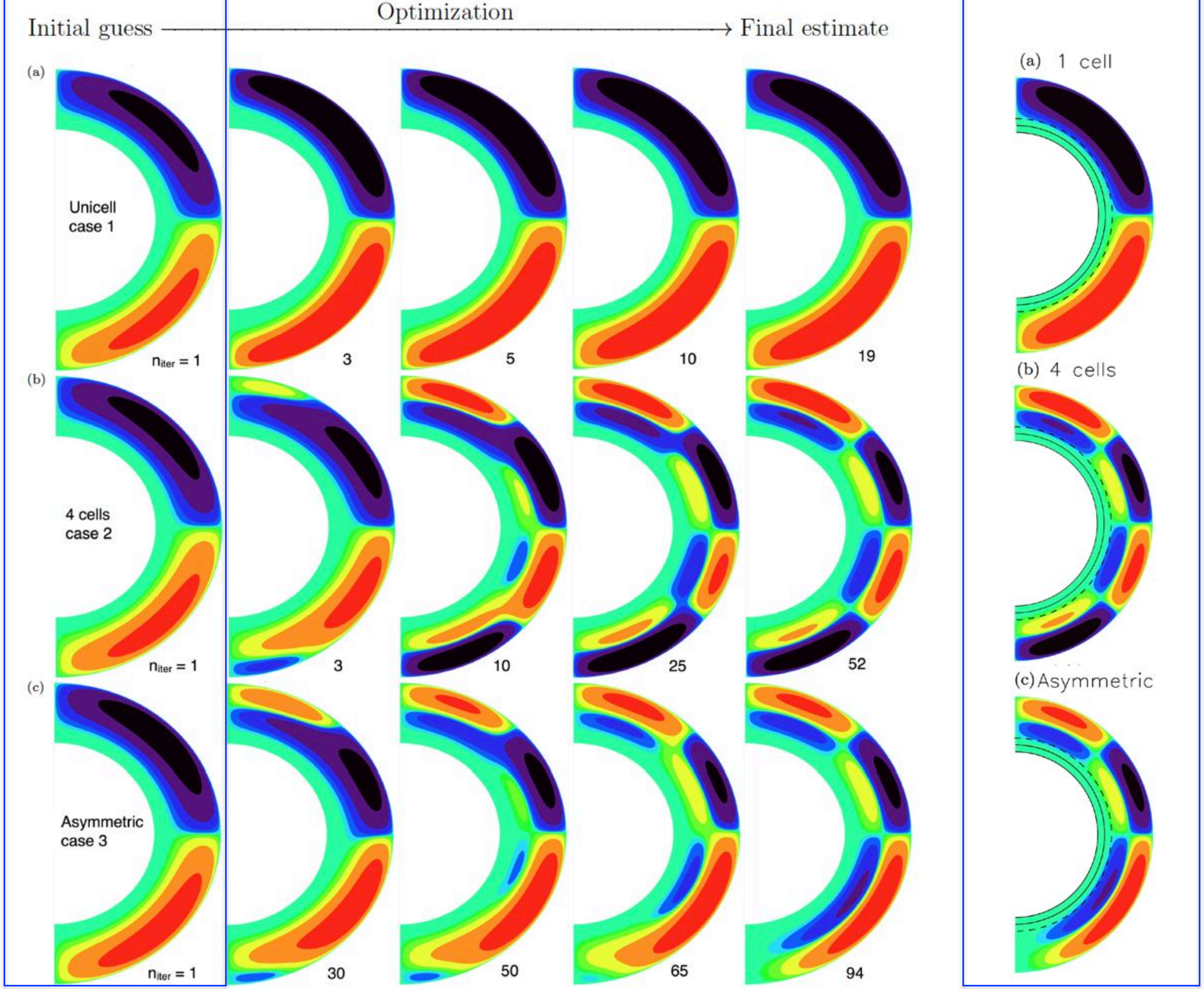
1 cell model

4 cell model

Asymmetric model

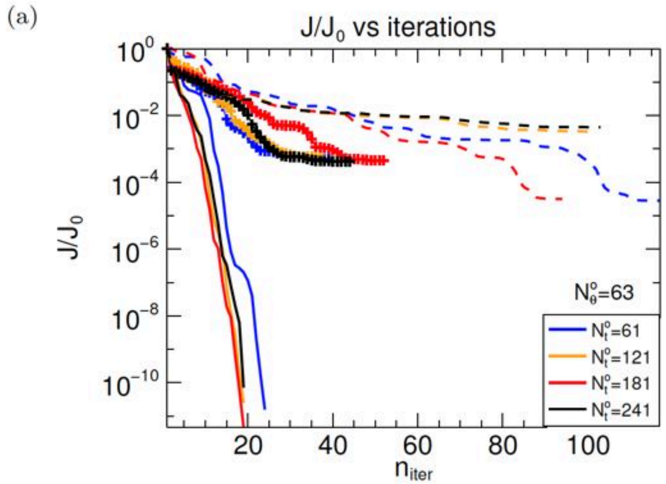
Initial guess

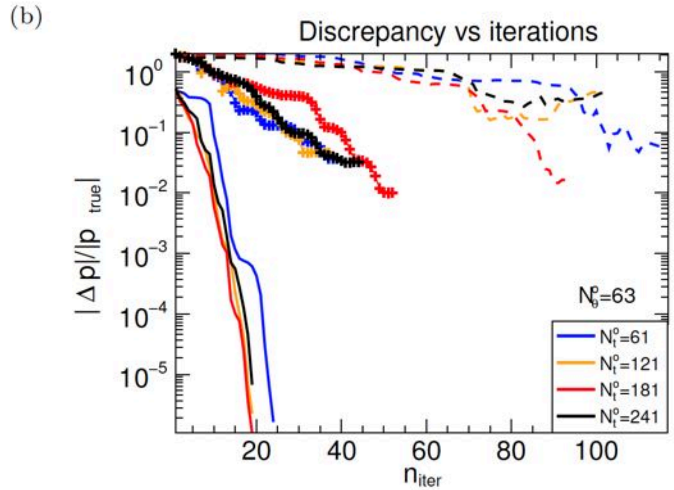
True state



4DVar in a dynamo model:
Results without noise

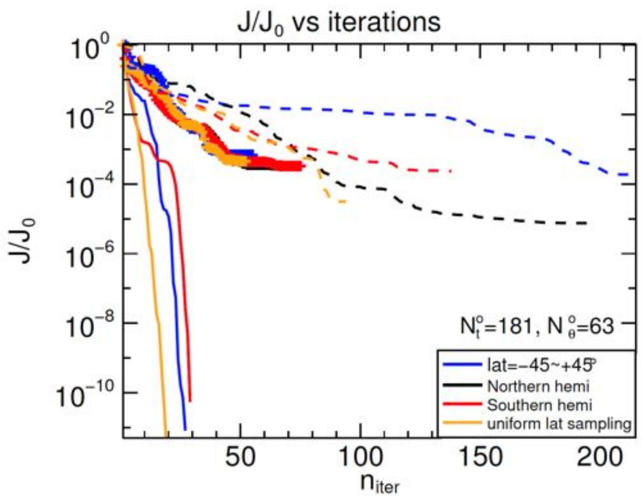
- Influence of the temporal window of assimilation

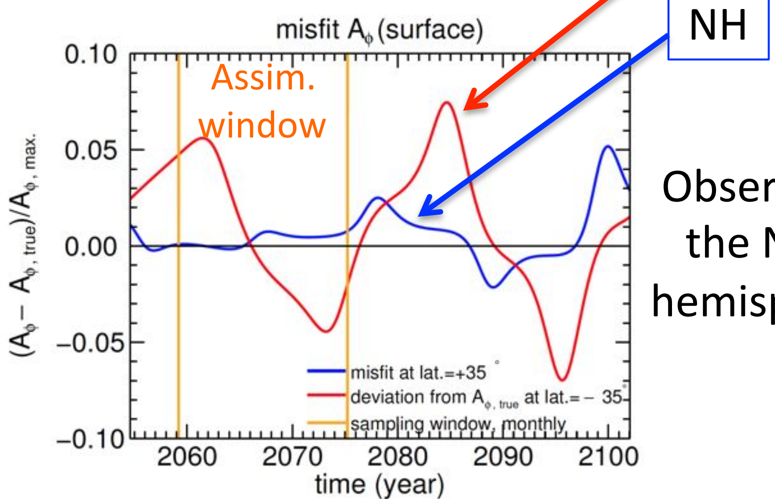




For all cases, a time window of 1.5 cycles seems to be optimal

- Influence of spatial distribution of observations

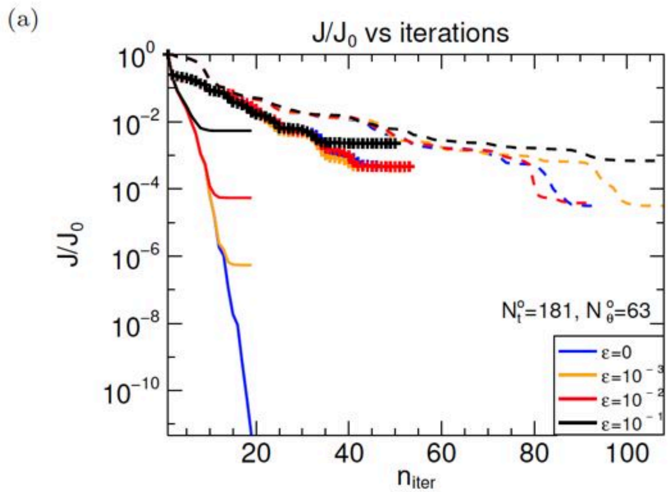


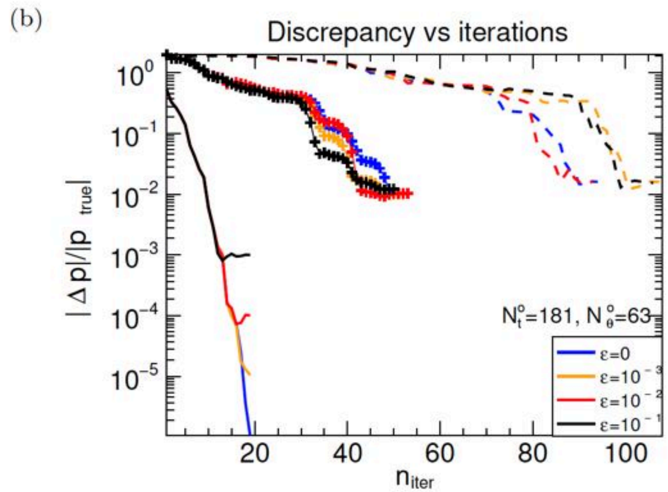


Observations in the Northern hemisphere only

4DVar in a dynamo model:
Results with noise

- ❑ Results for various noise levels, uniform sampling

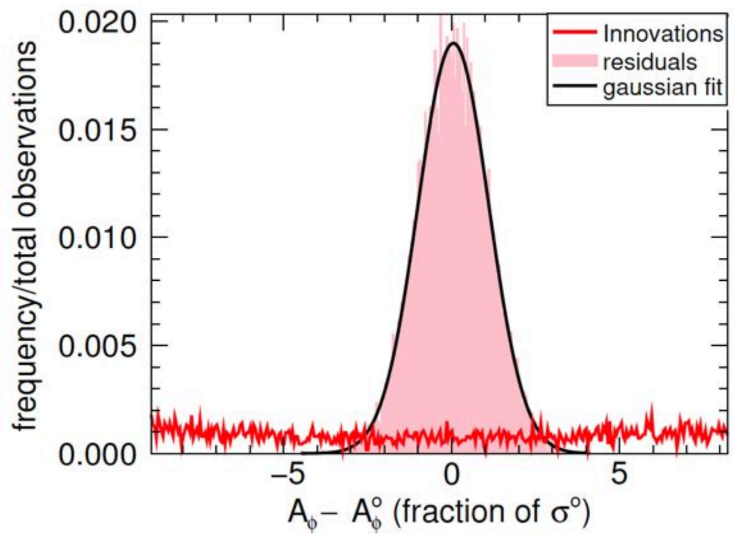




Example for
Cases 2 and 3
with 30% noise of
field recovery:
Aphi2
Aphi3

- ❑ Distribution of residuals after assimilation

Uniform sampling,
10% noise



Gaussian distribution
of residuals in agreement
with noise introduced

Normalized misfit close to 1

Conclusions and perspectives

- ❑ Data assimilation (DA) may be an interesting way to combine today's high quality observations and models/simulations

- ❑ DA may be used to infer potentially important ingredients of dynamo models: Hung et al. (2015) use a polar coordinate model which includes meridional circulation and recover both its amplitude and profile from noised magnetic data

- ❑ The model was used to produce the data (twin experiments), we now wish to move to real observations and actual predictions (for cycle 25?)

- ❑ Longer-term:

  Apply data assimilation techniques to a full spherical 3D MHD models of stars (many get large scale regular magnetic cycles now, e.g. Ghizaru, Brown, Augustson, Gastine, Käpylä, Warnecke, Hotta, Fan,…)
  and automatic differentiation algorithms exist to get the adjoint code!
  http://www-tapenade.inria.fr:8080/