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Title: The use of event-specific models in DREAM3D

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The use of event-specific models in DREAM3D

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Abstract

DREAM3D is a 3D Fokker-Planck diffusion code that has been used to model the dynamic evolution of MeV electrons in the radiation belts. The effects of drift-resonant ultra-low frequency (ULF) waves and gyro-resonant very-low frequency (VLF) waves, are modelled with quasilinear theory, which yields a 1D diffusion equation in dipole L at fixed values of the first and second invariants, and a 2D diffusion equation in pitch-angle and momentum at fixed L. The 1D and 2D diffusion equations are decoupled in DREAM3D because the background field is assumed to be a dipole and the 'cross-terms' are ignored. The diffusion coefficients are determined by the wave intensity in the ULF and VLF frequency ranges, and historically have been determined by statistical models for the wave intensity that depend on geomagnetic activity. Recently we have shown that the statistical models do not always perform well for a specific event, but 'event-specific' models that combine in-situ observations with the statistical models can be used to improve the model. For example, we have used measurements of the low-energy (~100 keV) population generated by the Van Allen Probes MagEIS instrument to define a low-energy boundary condition for DREAM3D as a function of time and L, and showed that modeling this 'seed population' correctly is critical for the model to predict the observed acceleration during the October 2012 storm. Similarly, we combined observations of the equatorial chorus wave intensity from the Van Allen Probes EMFISIS instrument, with precipitation observed by the NOAA POES instrument, to define an event-specific low-band chorus wave intensity. We showed that an event-specific model for the low-band chorus wave intensity is also critical for the model to predict the observed acceleration during the same storm. Our current efforts are aimed at extending our recent work on using event-specific models by incorporating non-dipole field models into DREAM3D for calculating more realistic radial and pitch-angle/momentum diffusion coefficients. In this talk, I will review our earlier results for the October 2012 storm using the dipole field model, present our approach to computing radial diffusion coefficients using the background field models developed by Tsyganenko and co-authors, and new results from DREAM3D using the new radial diffusion coefficients.

Outline

- Fokker-Planck codes vs MHD and PIC/Hybrid
- 'Standard' DREAM3D driven by statistical databases for wave amplitudes
- First major storm observed by Van Allen Probes- October 2012
- 'Event-specific' DREAM3D with time-varying boundary condition and wave amplitudes
- Recent work on including non-dipole effects, especially radial diffusion

Vlasov/MHD/PIC/Hybrid/Fokker-Planck

Solving general 2-way coupling (Vlasov equation) is difficult

$$f\left(\vec{x}, \vec{p}, t\right) \longleftrightarrow \vec{E}\left(\vec{x}, t\right), \vec{B}\left(\vec{x}, t\right)$$

Phase-space density (PSD)
particles/((vol)(momentum vol))

MHD
- Only calculate evolution of first two moments of PSD
- Can
  - simulate global geometry
  - couple particles and fields to produce ultra-low frequency (ULF) waves
  - produce large-amplitude fields that result in nonlinear transport
- Cannot
  - produce very low-frequency (VLF) waves
  - represent ‘tail’ of the energy distribution, i.e. MeV electrons, so these particles must be treated as ‘test particles’

Vlasov/MHD/PIC/Hybrid/Fokker-Planck

Solving general 2-way coupling (Vlasov equation) is difficult

$$f\left(\vec{x}, \vec{p}, t\right) \longleftrightarrow \vec{E}\left(\vec{x}, t\right), \vec{B}\left(\vec{x}, t\right)$$

Phase-space density (PSD)
particles/((vol)(momentum vol))

Particle-in-cell (PIC)/Hybrid

- Represent PSD with weighted particles or particles+fluid
- Can
  - produce VLF waves when distribution unstable to wave growth
  - produce nonlinear transport of test particles: MeV e- due to large wave amplitudes
- Cannot simulate global system due to computational load
  - produce very low-frequency (VLF) waves
  - represent 'tail' of the energy distribution, i.e. MeV electrons, so these particles must be treated as 'test particles'

Vlasov/MHD/PIC/Hybrid/Fokker-Planck

Solving general 2-way coupling (Vlasov equation) is difficult

$$f\left(\vec{x},\vec{p},t\right) \longleftrightarrow \vec{E}\left(\vec{x},t\right),\vec{B}\left(\vec{x},t\right)$$

Phase-space density (PSD)
particles/((vol)(momentum vol))

Fokker/Planck

- Treat full PSD with *specified* fields: static background, stochastic time-varying perturbations
- Average evolution of PSD is computed with diffusion equation
- Can
  - simulate global system with low computational burden
  - treat MeV electrons as part of the full distribution
- Cannot
  - two-way couple particle distributions and fields
  - represent nonlinear transport due to small amplitude of perturbations

Vlasov/MHD/PIC/Hybrid/Fokker-Planck

Solving general 2-way coupling (Vlasov equation) is difficult

$$f\left(\vec{x}, \vec{p}, t\right) \longleftrightarrow \vec{E}\left(\vec{x}, t\right), \vec{B}\left(\vec{x}, t\right)$$

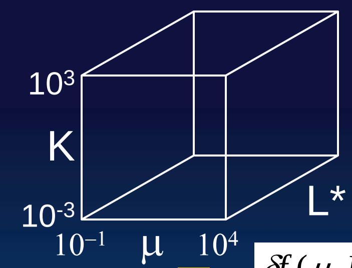
Phase-space density (PSD)
particles/((vol)(momentum vol))

MHD

Fokker-Planck

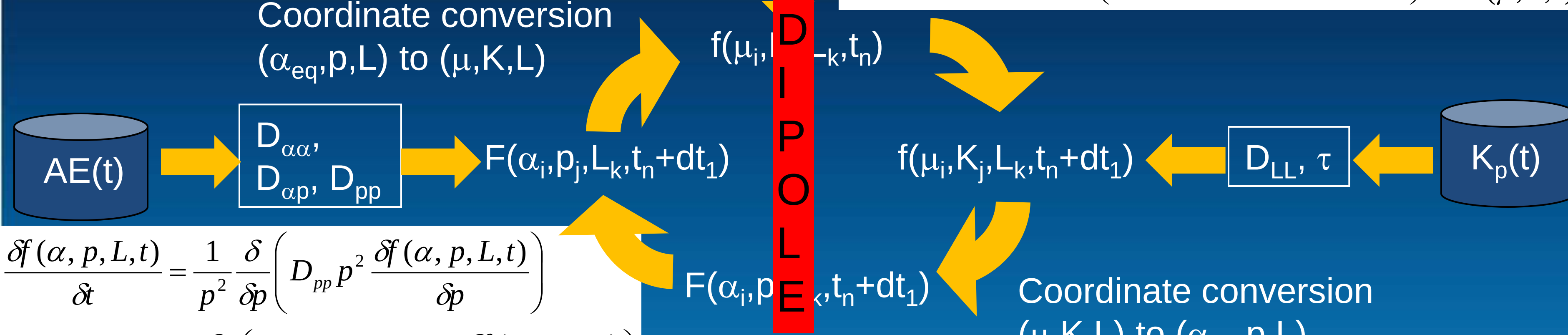
PIC/Hybrid

‘Standard’ DREAM3D is a 3D diffusion code implemented as 1D+2D diffusion (AGU 2012)



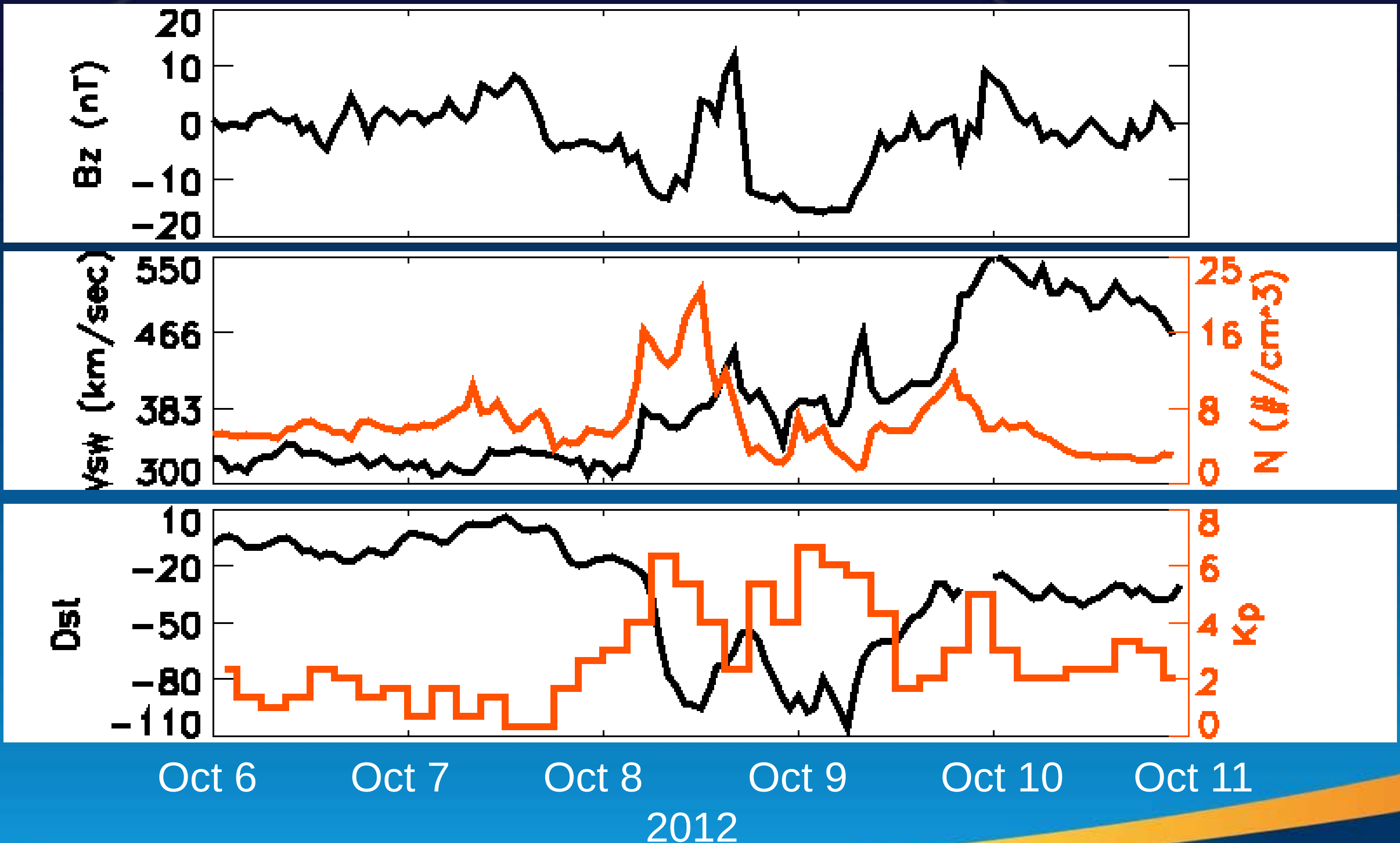
Initial condition uses TS04 to create $f(\mu,K,L^*,t_0)$

$$\frac{\delta f(\mu,K,L,t)}{\delta t} = L^2 \frac{\delta}{\delta L}\left(\frac{D_{LL}(\mu,L,t)}{L^2}\frac{\delta f(\mu,K,L,t)}{\delta L}\right) - \frac{f(\mu,K,L,t)}{\tau(\mu,L,t)}$$

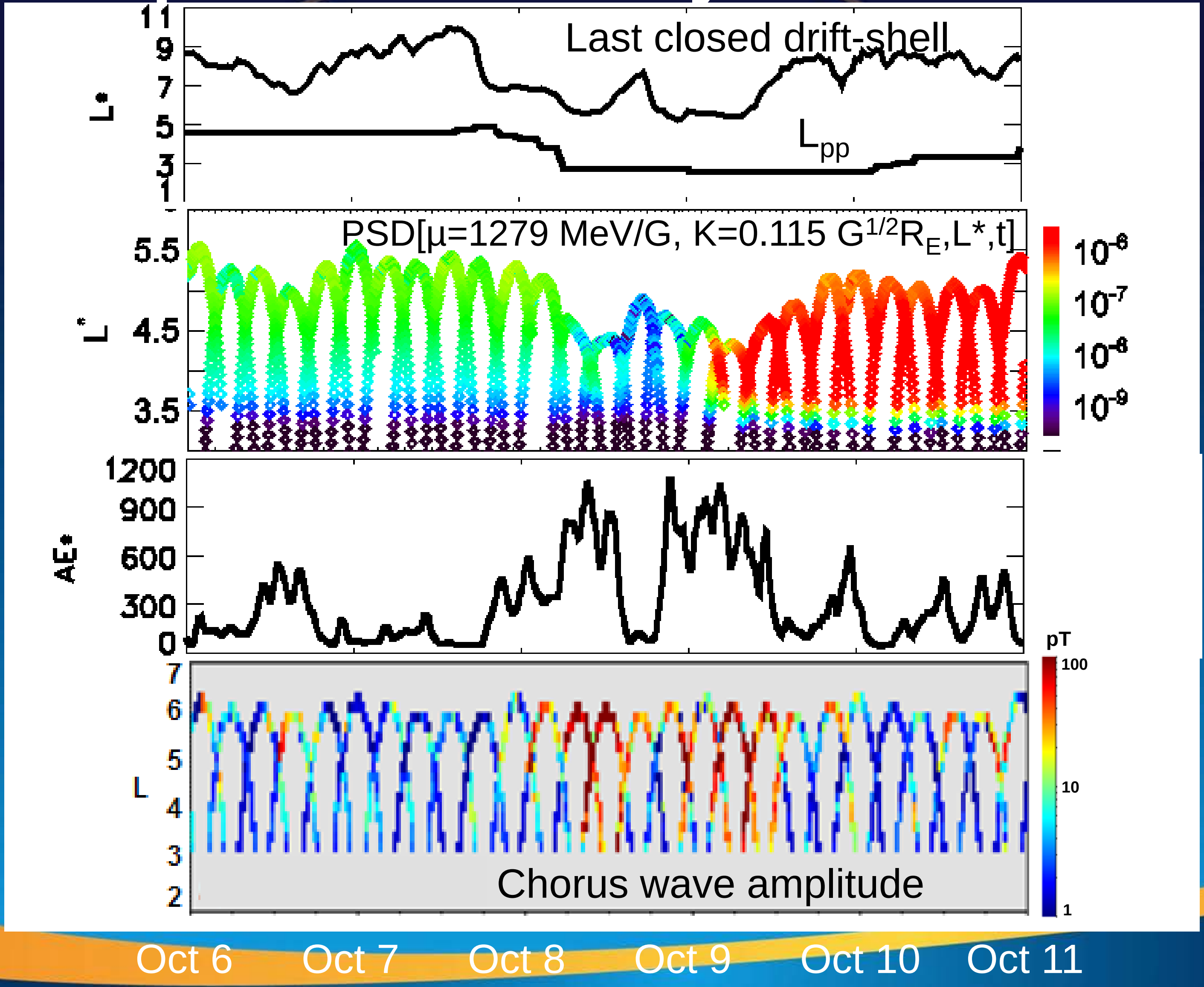


$$\frac{\delta f(\alpha,p,L,t)}{\delta t} = \frac{1}{p^2}\frac{\delta}{\delta p}\left(D_{pp}p^2\frac{\delta f(\alpha,p,L,t)}{\delta p}\right) + \frac{1}{T(\alpha)\sin(2\alpha)}\frac{\delta}{\delta\alpha}\left(D_{\alpha\alpha}T(\alpha)\sin(2\alpha)\frac{\delta f(\alpha,p,L,t)}{\delta\alpha}\right) - \frac{f(\alpha,p,L,t)}{\tau(\alpha,L)}$$

October 2012 storm: a CME 'grazed' earth and caused a 'double-dip' event

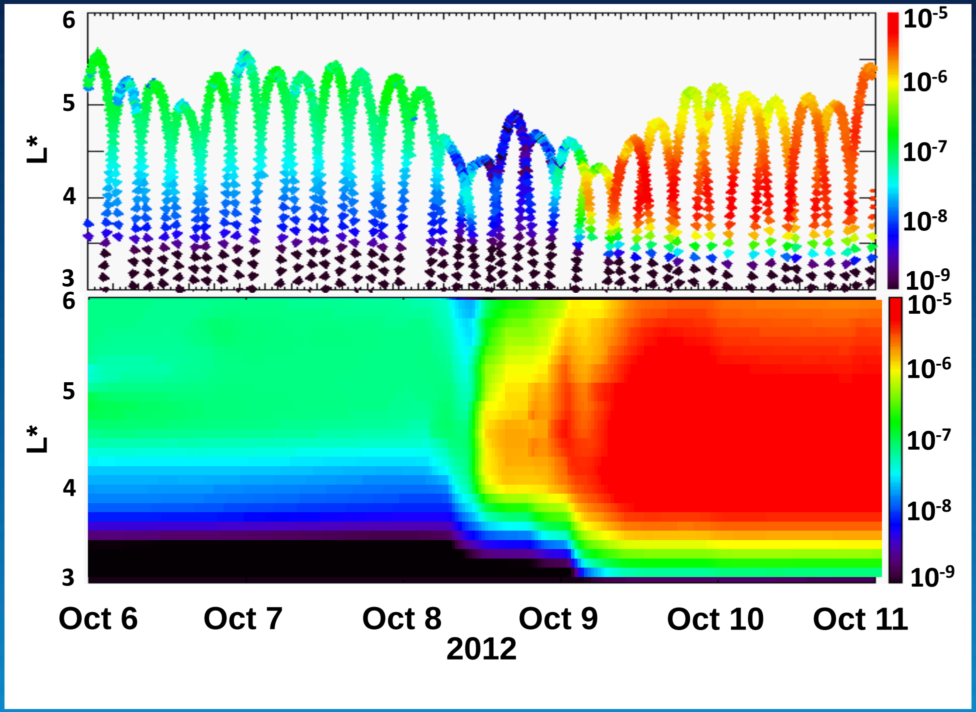


A dropout followed by enhancement



‘Standard’ DREAM3D over-estimates late acceleration; no early dropout

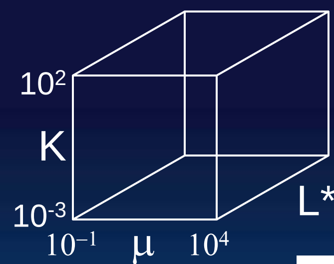
PSD[μ=1279 MeV/G, K=0.115 $G^{1/2}R_E$,L*,t]



Van Allen Probes electron data (MagEIS, REPT)

‘Standard’ DREAM3D

‘Event-specific’ DREAM3D (AGU 2013)



Initial condition uses TS04 to create $f(\mu,K,L^*,t_0)$

$$\frac{\delta f(\mu,K,L,t)}{\delta t} = L^2 \frac{\delta}{\delta L}\left(\frac{D_{LL}(\mu,L,t)}{L^2}\frac{\delta f(\mu,K,L,t)}{\delta L}\right) - \frac{f(\mu,K,L,t)}{\tau(\mu,L,t)}$$

POES proxy for LB chorus

AE(t)

Coordinate conversion (α_{eq},p,L) to (μ,K,L)

$D_{\alpha\alpha}$, $D_{\alpha p}$, D_{pp}

$F(\alpha_i,p_j,L_k,t_n+dt_1)$

$f(\mu_i,K_j,L_k,t_n)$

DIPOLE

$f(\mu_i,K_j,L_k,t_n+dt_1)$

D_{LL}, τ

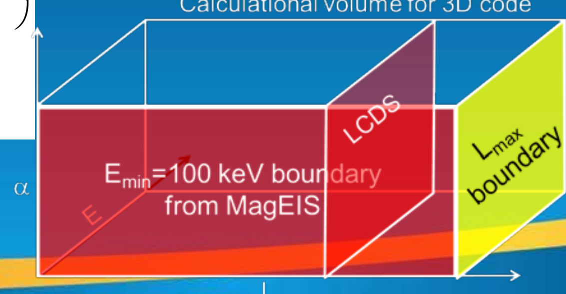
$K_p(t)$

$$\frac{\delta f(\alpha,p,L,t)}{\delta t} = \frac{1}{p^2}\frac{\delta}{\delta p}\left(D_{pp}p^2\frac{\delta f(\alpha,p,L,t)}{\delta p}\right) + \frac{1}{T(\alpha)\sin(2\alpha)}\frac{\delta}{\delta\alpha}\left(D_{\alpha\alpha}T(\alpha)\sin(2\alpha)\frac{\delta f(\alpha,p,L,t)}{\delta\alpha}\right) - \frac{f(\alpha,p,L,t)}{\tau(\alpha,L)}$$

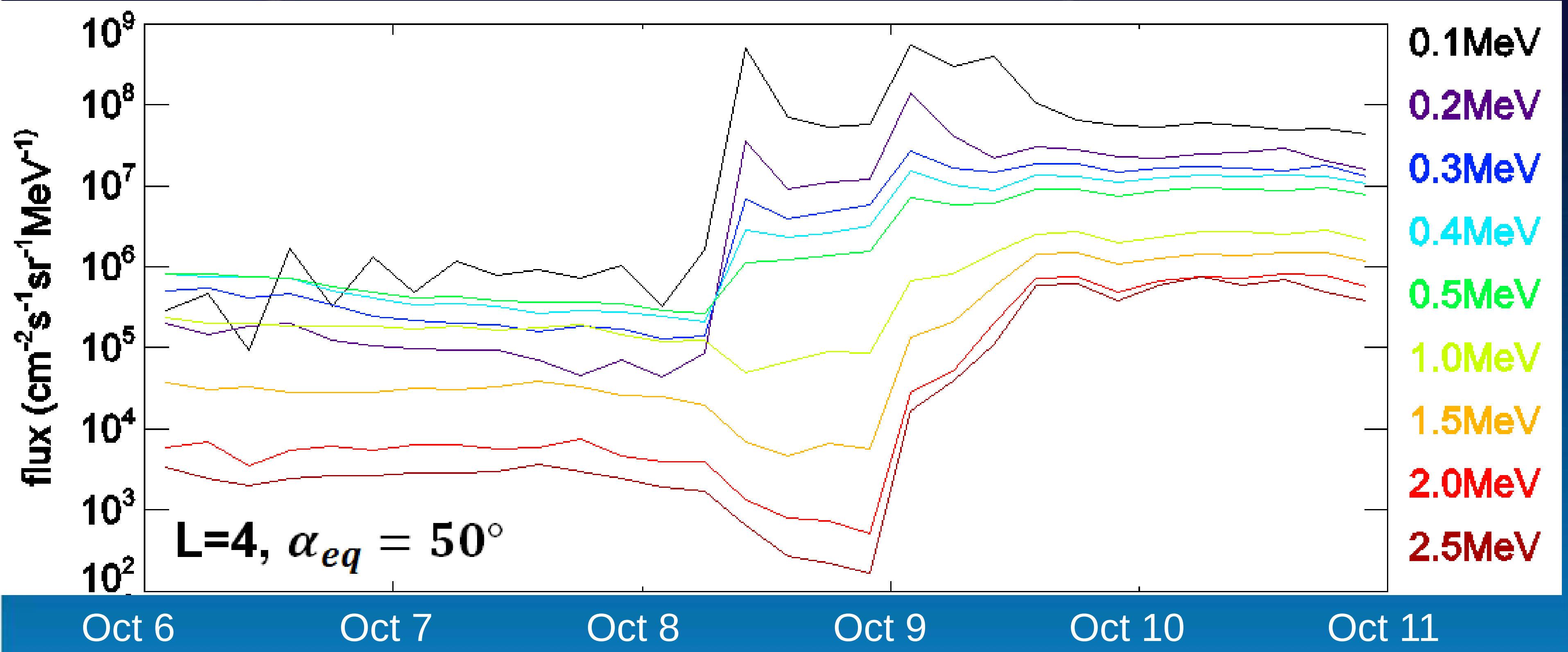
$F(\alpha_i,p_j,L_k,t_n+dt_1)$

Coordinate conversion (μ,K,L) to (α_{eq},p,L)

Calculational volume for 3D code



E_{min}=100 keV boundary condition supplies dynamic 'seed' population

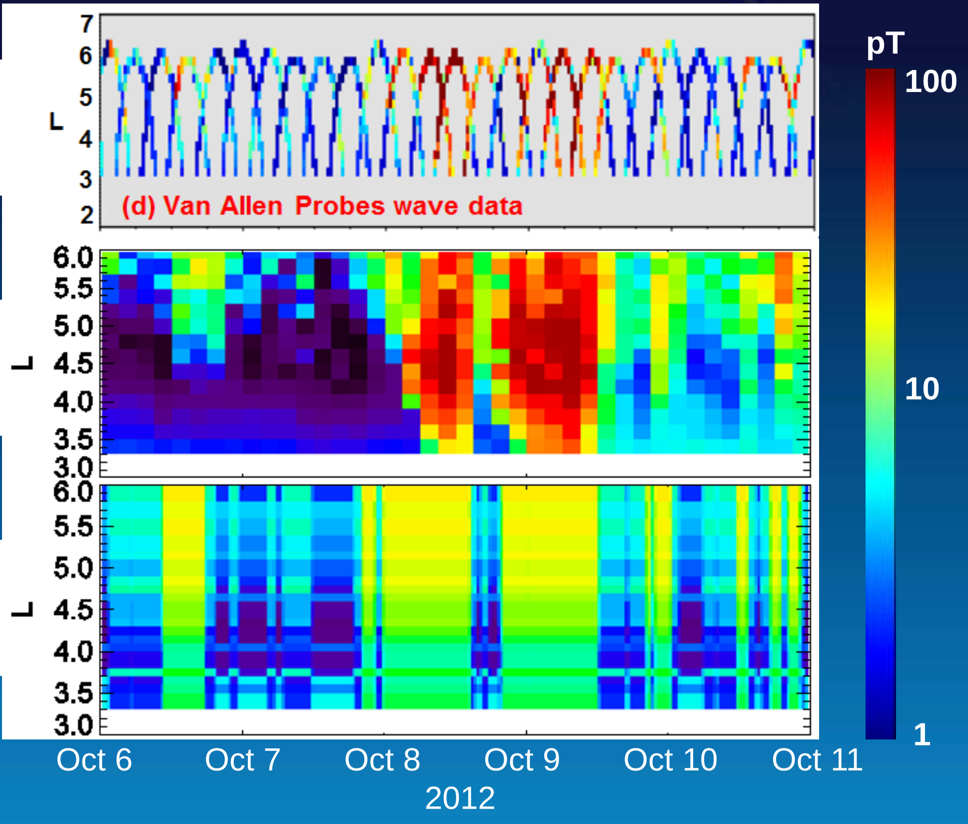


Event-specific chorus wave intensities much higher than statistical database

Van Allen Probes wave msmts

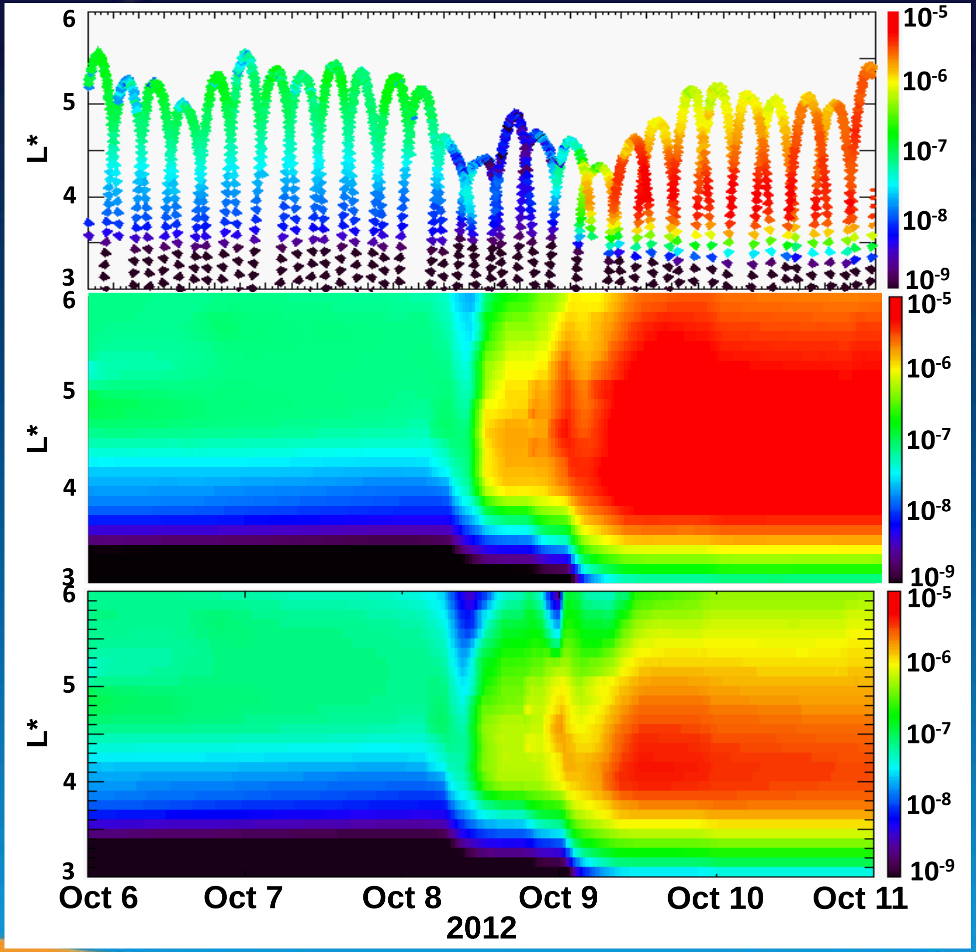
POES precipitation wave proxy

AE*-binned statistical database



'Event-specific' DREAM3D reproduces late enhancement but not early dropout

PSD[μ=1279 MeV/G, K=0.115 $G^{1/2}R_E$,L*,t]



Van Allen Probes electron data (MagEIS, REPT)

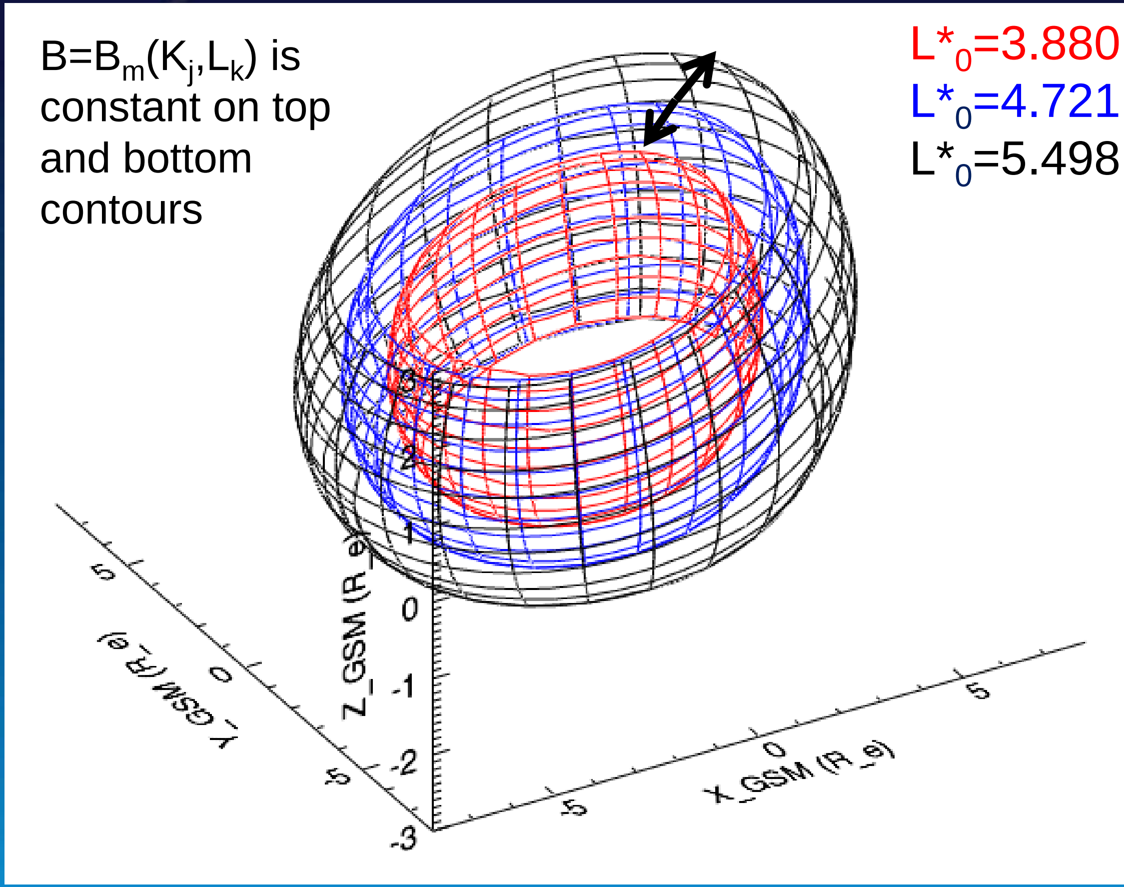
'Standard' DREAM3D

'Event-specific' DREAM3D w/ event-specific E_{min} BC, LCDS, and lowband chorus wave intensity

Major problem with ‘Standard’ and ‘Event-specific’ DREAM3D

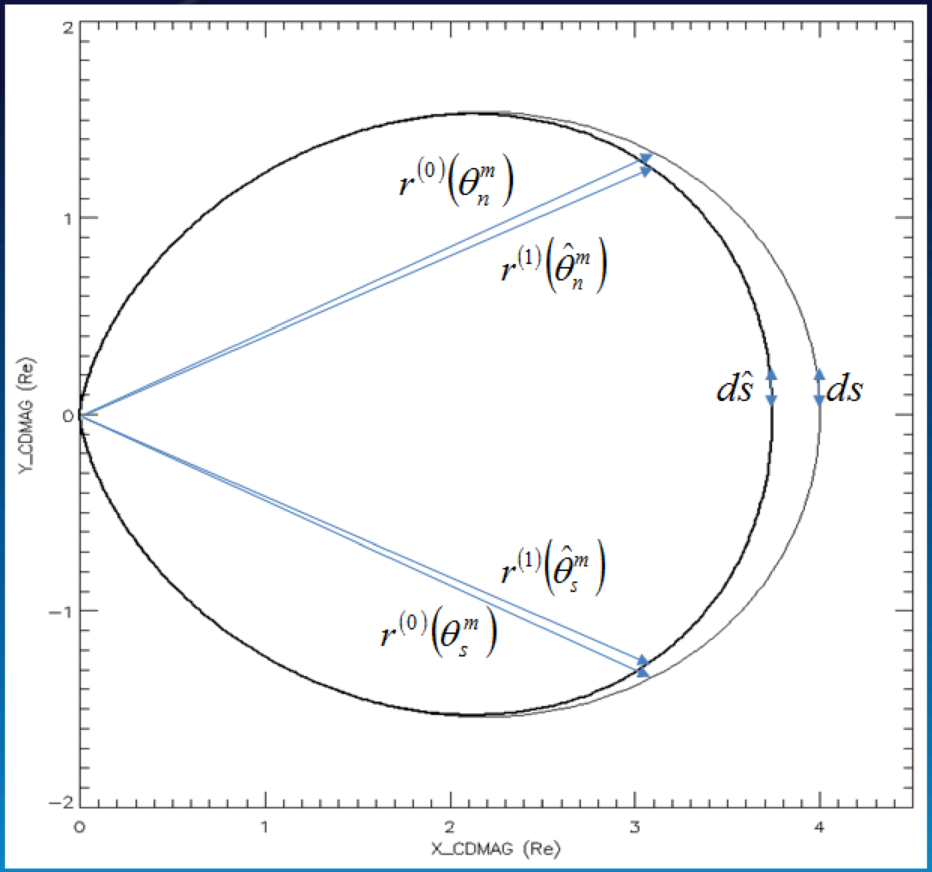
- Dipole field is used throughout model
  - Brautigam and Albert 2000 model of D_{LL} based on Falthammar 1965 and Schulz and Lanzerotti 1974
  - Conversion from μ, K, L* to α, p, L
  - Evaluation of pitch-angle/energy diffusion coefficients $D_{\alpha\alpha}$, $D_{\alpha p}$, D_{pp}
- Solution: incorporate Tsyganenko empirical field models throughout code

A general approach to radial diffusion: small-amplitude dynamic perturbation field



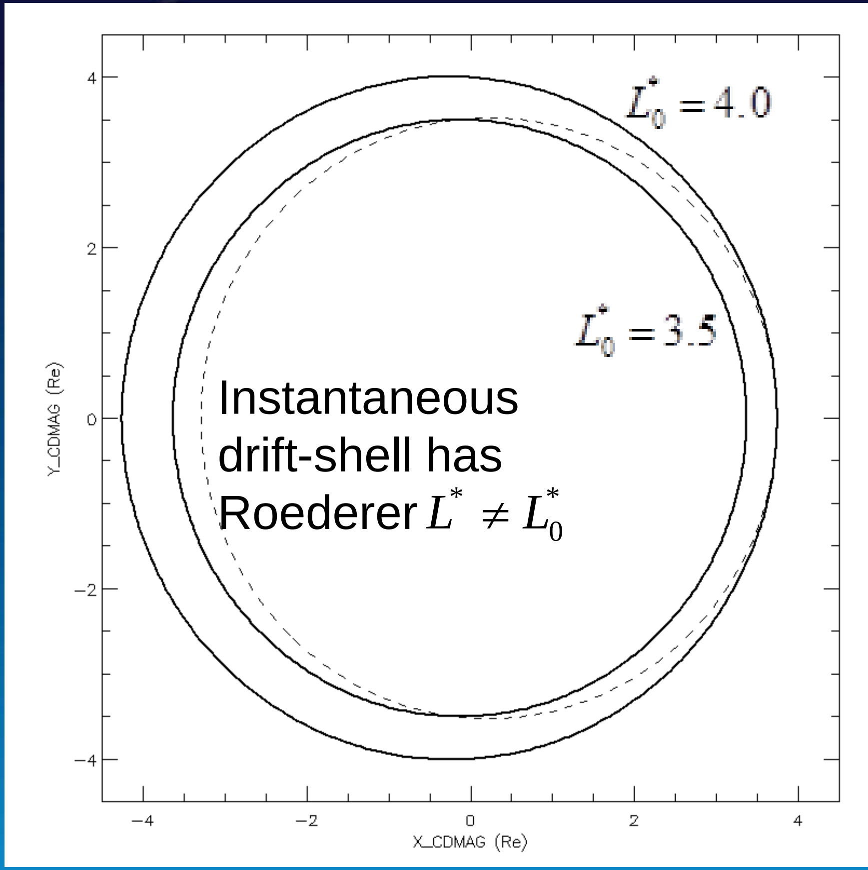
A small-amplitude magnetic perturbation, when added to the background field, causes particles to move from field lines associated with one drift-shell in the unperturbed field to another.

A general approach to radial diffusion: individual field lines are perturbed



At a given azimuth, the magnetic perturbation changes the geometry of a field line associated with a given drift-shell. This also changes the dependence of K on B_m.

A general approach to radial diffusion: movement of K=0 particles across L^*_0



Solid lines plot the perturbed field line locations of the K=0 drift shells. Ionospheric footpoints of perturbed field lines are unchanged. Gradient-curvature drift (dashed line) causes change in L^*_0 of field line as the particle moves. The rate of change is dL^*_0/dt.

$$D_{L_0^* L_0^*} = \lim_{\Delta t \to \infty} \frac{E\left[\left(\Delta L_0^*\right)^2\right]}{2\Delta t}$$

A general approach to radial diffusion: how do we calculate dL^*_0/dt?

$$\frac{d\Delta L_0^*}{dt} = \frac{\Delta L_0^*}{\Delta\varphi}\frac{d\varphi}{dt}$$

$dL^*_0/d\varphi=0$ with no perturbation since the particles stay on field lines associated with a fixed drift-shell. The $d\varphi/dt$ in the unperturbed field is all that counts here.

$$dK = \left(\delta K/\delta L_0^*\right)dL_0^* + \left(\delta K/\delta\varphi\right)d\varphi = 0$$

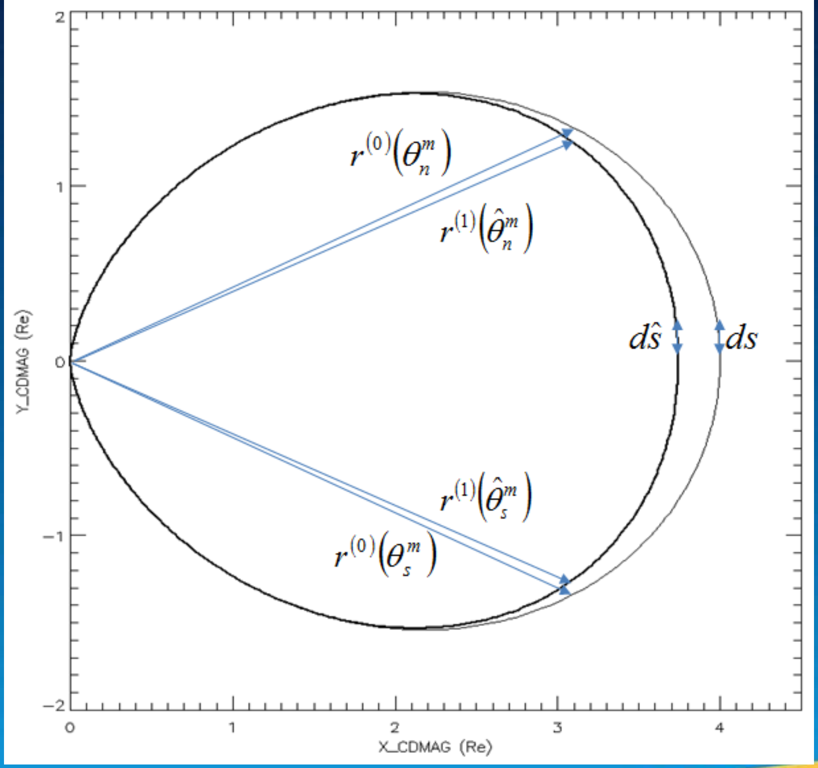
$$\downarrow$$

$$\frac{dL_0^*}{d\varphi} = -\frac{\delta K/\delta\varphi}{\delta K/\delta L_0^*}$$

In the perturbed field, K at fixed B_m conserved as the particle drifts.

A general approach to radial diffusion: second invariant in the perturbed field

$$\hat{K} = \int_{\hat{\theta}_m^s}^{\hat{\theta}_m^n} \sqrt{B_m - \left|\vec{B}\left(r^{(1)}(\theta,\varphi),\varphi^{(1)}(\theta,\varphi)\right) + \vec{b}\left(r^{(1)}(\theta,\varphi),\varphi^{(1)}(\theta,\varphi)\right)\right|}\,\frac{d\hat{s}}{d\theta}\,d\theta$$



Given K, the azimuth, φ, and drift-shell, L^*_0, specify a unique field line that is perturbed. The value of K is also perturbed.

A general approach to radial diffusion: the quasilinear theory for arbitrary background

(13)

$$\hat{K} \sim \int_{\hat{\theta}_m^s}^{\hat{\theta}_m^n} \sqrt{f(\theta_m^s) - f(\theta)} \sqrt{1 - \frac{g(\theta)}{f(\theta_m^s) - f(\theta)} \left(\frac{ds}{d\theta} + h(\theta) \right)} d\theta$$

$$\sim K + \int_{\theta_m^s}^{\theta_m^n} \sqrt{f(\theta_m^s) - f(\theta)} h(\theta) d\theta - \frac{1}{2} \int_{\theta_m^s}^{\theta_m^n} \frac{g(\theta)}{\sqrt{f(\theta_m^s) - f(\theta)}} \frac{ds}{d\theta} d\theta \sim K + \Delta K_1 + \Delta K_2$$

$$f(\theta,\varphi) = \left| \vec{B}\left(r^{(0)}(\theta,\varphi), \varphi^{(0)}(\theta,\varphi) \right) \right|$$

$$g(\theta,\varphi) = \left| \vec{B}\left(r^{(1)}(\theta,\varphi), \varphi^{(1)}(\theta,\varphi) \right) + \vec{b}\left(r^{(1)}(\theta,\varphi), \varphi^{(1)}(\theta,\varphi) \right) \right| \sim B + b_{\|} + \frac{\delta B}{\delta r} \Delta r + \frac{\delta B}{\delta \varphi} \Delta \varphi \Big|_{r^{(0)}(\theta,\varphi),\varphi^{(0)}(\theta,\varphi)}$$

$$\frac{d\hat{s}}{d\theta} \sim \frac{ds}{d\theta} \left[1 + \left(\frac{ds}{d\theta} \right)^{-2} \left\{ \begin{array}{l} r^{(0)}(\theta,\varphi) \Delta r(\theta,\varphi) + \dfrac{dr^{(0)}(\theta,\varphi)}{d\theta} \dfrac{d\Delta r(\theta,\varphi)}{d\theta} \\ + \left\{ r^{(0)}(\theta,\varphi) \sin(\theta) \right\}^2 \dfrac{d\varphi^{(0)}(\theta,\varphi)}{d\theta} \dfrac{d\Delta\varphi(\theta,\varphi)}{d\theta} \\ + r^{(0)}(\theta,\varphi) \sin^2(\theta) \Delta r(\theta,\varphi) \left\{ \dfrac{d\varphi^{(0)}(\theta,\varphi)}{d\theta} \right\}^2 \end{array} \right\} \right] = \frac{ds}{d\theta} + h(\theta,\varphi)$$

A general approach to radial diffusion: numerical evaluation of coefficients

- Previous slide quantifies ΔK as a function of φ, L^*_0, for each drift shell (which specifies K, Bm) so that the needed derivatives in this equation can be numerically evaluated:

$$\frac{dL_0^*}{d\varphi} = -\frac{\delta K \big/ \delta\varphi}{\delta K \big/ \delta L_0^*}$$

A general approach to radial diffusion: additional work to get $D_{L^*L^*}$

- For a particle starting at (K, L*, ϕ_0)

$$\Delta L_0^* = \int_0^t \frac{d\Delta L_0^*\left(K, L_0^*, \phi(t')\right)}{dt} dt' = \int_0^t f\left(K, L_0^*, \phi(t')\right) A(t') \Omega_d(\phi(t')) dt'$$

$$\phi(t) = \phi_0 + \int_0^t \Omega_d(\phi(t')) dt'$$

- The drift period, T, where $2\pi = \int_0^T \Omega_d(\phi(t')) dt'$ plays a critical role since

$$g(t) = f\left(K, L_0^* *, \phi(t)\right) \Omega_d(\phi(t))$$

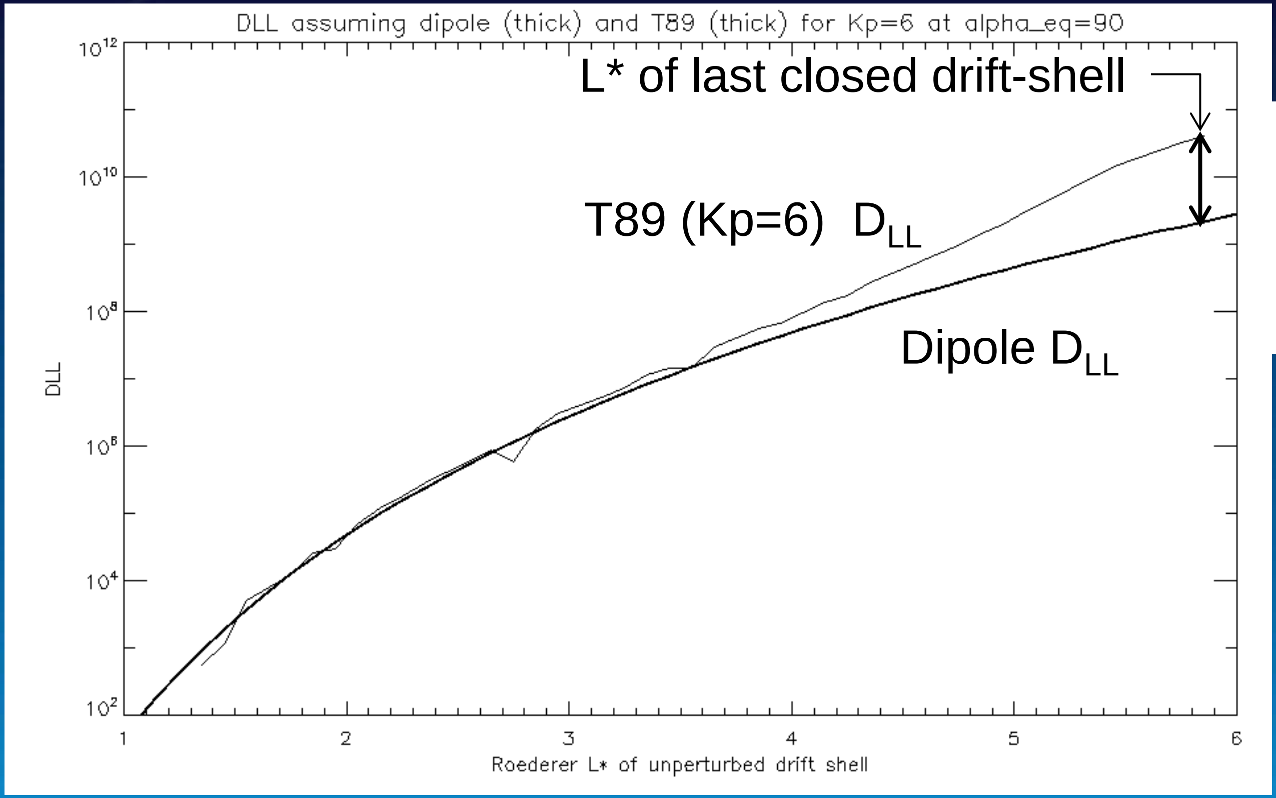
is periodic with period T $g(t+T) = g(t)$

A general approach to radial diffusion: additional work to get $D_{L^*L^*}$

- Since g(t) is periodic we have $g(t)=\sum_{n=-\infty}^{\infty} g_n e^{jn2\pi/T}$
- The diffusion coefficient samples the power spectrum of A(t) at harmonics of the drift frequency

$$
\begin{aligned}
D_{L_0^*L_0^*} &= \lim_{t\to\infty}\frac{E\left[\left(\Delta L_0^*\right)^2\right]}{2t} = \lim_{t\to\infty}\frac{1}{2t}\int_0^t\int_0^t \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g_n^* g_m e^{j2\pi m t_1/T} e^{-j2\pi m t_2/T} R_A(t_1-t_2)\,dt_1 dt_2 \\
&= \lim_{t\to\infty}\frac{1}{2t}\int_0^{2t}\int_{-u}^{u} \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g_n^* g_m e^{j2\pi(m-n)u/2T} e^{j2\pi(m+n)\tau/2T} R_A(\tau)\,d\tau du \\
&= \sum_{m=-\infty}^{\infty} \left|g_m\right|^2 S_A\left(\frac{2\pi m}{T}\right)
\end{aligned}
$$

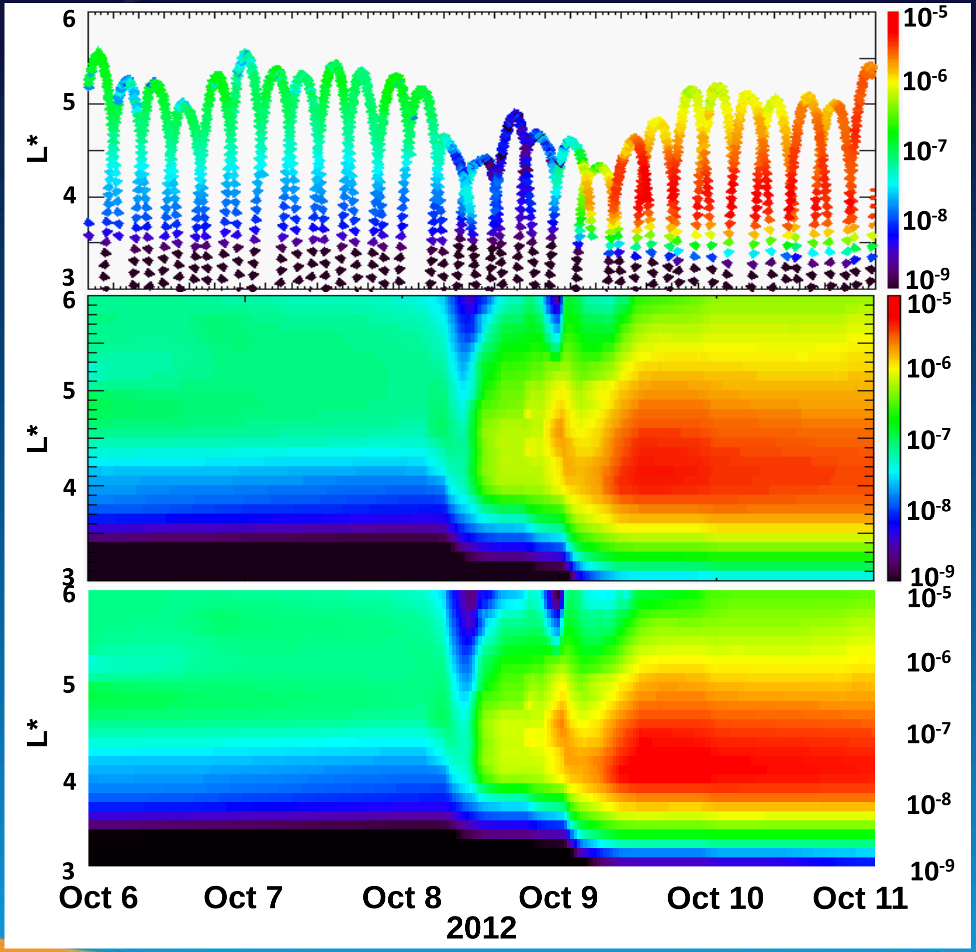
DLL using T89 for background field much larger for large L* during high Kp



More than an order of magnitude larger DLL near LCDS using T89 with Kp=6.

DREAM3D with radial diffusion in T89 background field improves dropout

PSD[μ=1279 MeV/G, K=0.115 $G^{1/2}R_E$,L*,t]



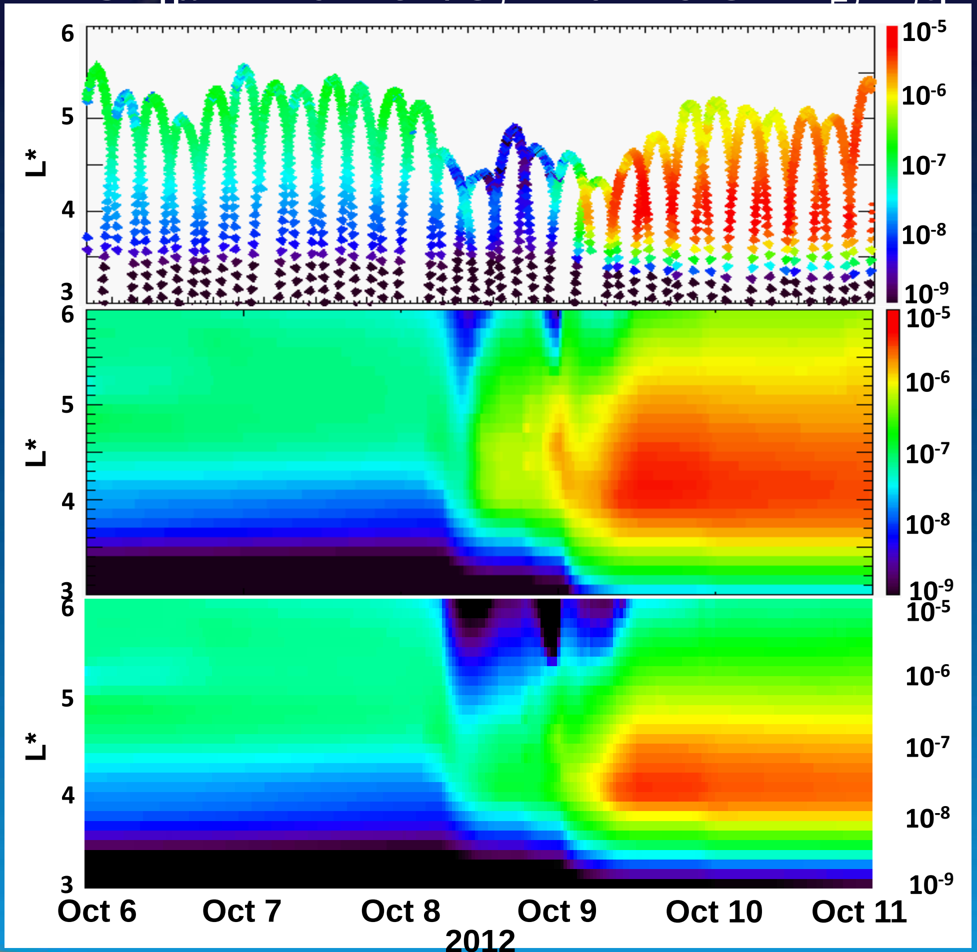
Van Allen Probes electron data (MagEIS, REPT)

'Event-specific' DREAM3D w/ event-specific E_{min} BC, LCDS from TS04, and lowband chorus wave intensity

'Event-specific' DREAM3D w/DLL from T89 and LCDS from TS04

DREAM3D with radial diffusion in a non-dipole field reproduces deep dropout

PSD[μ=1279 MeV/G, K=0.115 $G^{1/2}R_E$,L*,t]



Van Allen Probes electron data (MagEIS, REPT)

'Event-specific' DREAM3D w/ event-specific E_{min} BC, LCDS from TS04, and lowband chorus wave intensity

'Event-specific' DREAM3D w/DLL from T89 and LCDS from TS04 –lifetime outside LCDS is zero

Conclusion and discussion

- Event-specific boundary condition, waves and DLL all improve model for this complex event
- Effect of increased outward radial diffusion using T89 background field is comparable to variability in ULF wave power at fixed Kp
  – Drift-shell inflation increases D_{LL} due to larger perturbation and smaller background field
  – Azimuthal variation in drift velocity decreases D_{LL} due to assumption of spectral form $1/f^2$
- Treatment of particles outside the LCDS important

Future work

- More realistic background field (TS04, TS07?)
- Use of T89/TS04/TS07 field in
  - coordinate transformations
  - diffusion coefficients for VLF
  - ‘mixed’ diffusion $D_{\alpha L}$
- Event-specific plasma density, ULF wave intensity/spectrum/distribution